Direct numerical modelling of the transient convectional flow

1. Formulation of Problem.

We use model Boussinesq and guess a fluid viscous and incompressible:

$$\begin{cases} \omega_t + 1/Pr\,(\varphi_y\,\omega_x - \varphi_x\,\omega_y) = \Delta\omega + Ra\ Q_x, \\ \Delta\varphi = -\,\omega, \\ Q_t + 1/Pr\,(\varphi_y\,Q_x - \varphi_x\,Q_y) = 1/Pr\ \Delta Q - 1/Pr\ \varphi_x, \end{cases}$$

Here ω- vortex, φ - stream function, Q – temperature's diversion from an equilibrium distribution, $\Theta = 1 - y + Q$ - complete temperature. $Ra = g\beta H^3 dQ/\chi\nu$ - Raleigh's number, $Pr = \nu/\chi$- Prandtl's number, g – acceleration due to gravity, β - thermal expansion coefficient, ν - kinematics viscosity, χ - heat diffusivity, H – width of a layer.

The convection of the Raleigh - Bernard is considered. The horizontal boundaries are guessed flat, isothermal and free from shearing stresses. The flow arises at heating from below. The equations are written in diversions from the equilibrium solution. Equilibrium solution is unstable for

r   1, $r = Ra/Ra_{cr}$, $Ra_{cr} = 657.5$.

$$\varphi = \omega = Q = 0$$

$$\varphi = \omega = Q = 0$$

We considered the solutions in a spectral view:

$$\omega(t,x,y) = \sum \omega_{km}(t)\cos(\alpha kx)\sin(\pi my),$$
$$\varphi(t,x,y) = \sum \varphi_{km}(t)\cos(\alpha kx)\sin(\pi my),$$
$$Q(t,x,y) = \sum Q_{km}(t)\sin(\alpha kx)\sin(\pi my).$$

Here $\alpha = \pi / L$ – wave number, and L - length area divided on width layer, N - number x-harmonics, M - number y-harmonics.

2. The numerical method.

I.B. Palymsky. Method for numerical modelling of convective flows. Numerical technologies, vol. 5, № 6, pp. 53 – 61(2000). Novosibirsk, Russia (in Russian).

We use split method on linear and nonlinear processes.

<u>Step 1.</u> Linear development of perturbations.

$$\begin{cases} \omega_t = \frac{1}{2}\Delta\omega + RaQ_x, \\ \Delta\varphi = -\omega, \\ Q_t = 1/(2Pr)\Delta Q - 1/Pr\,\varphi_x. \end{cases}$$

This system is solved ***analytically*** in spectral space, without numerical methods.

<u>Step 2.</u> Nonlinear convective transport.

$$\begin{cases} \omega_t + 1/Pr(\varphi_y\omega_x - \varphi_x\omega_y) = 1/2\Delta\omega, \\ Q_t + 1/Pr(\varphi_y Q_x - \varphi_x Q_y) = 1/(2Pr)\Delta Q. \end{cases}$$

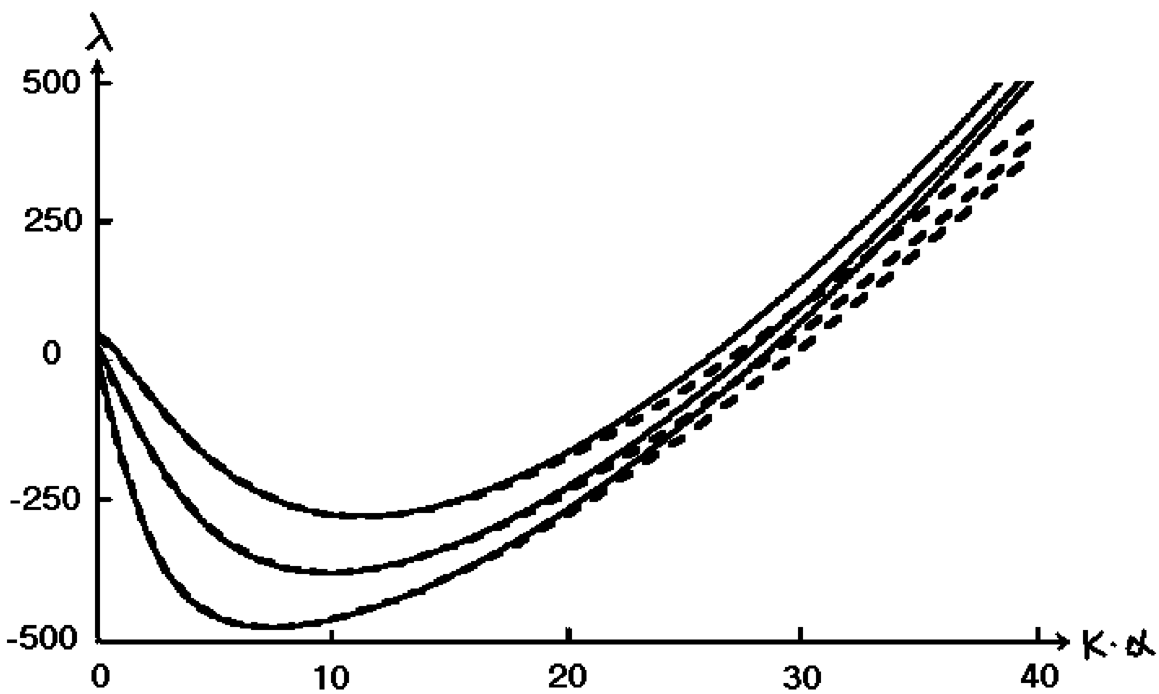
We solved this system ***numerically*** by alternating direction method in physical space. For approximation derivative we utilized the finite differences.

The order of approximation of the numerical method is Î$(\tau + H^2)$.

As a rule, Pr=2, α =1, N = 65, M = 17, $\tau = 4*10^{-5}$ - step on the time.

To test precision of the numerical method, we test reproduction infinitesimal of perturbations (linear analysis) for a differential problem and numerical method. We consider the solutions of linear problems in a view:

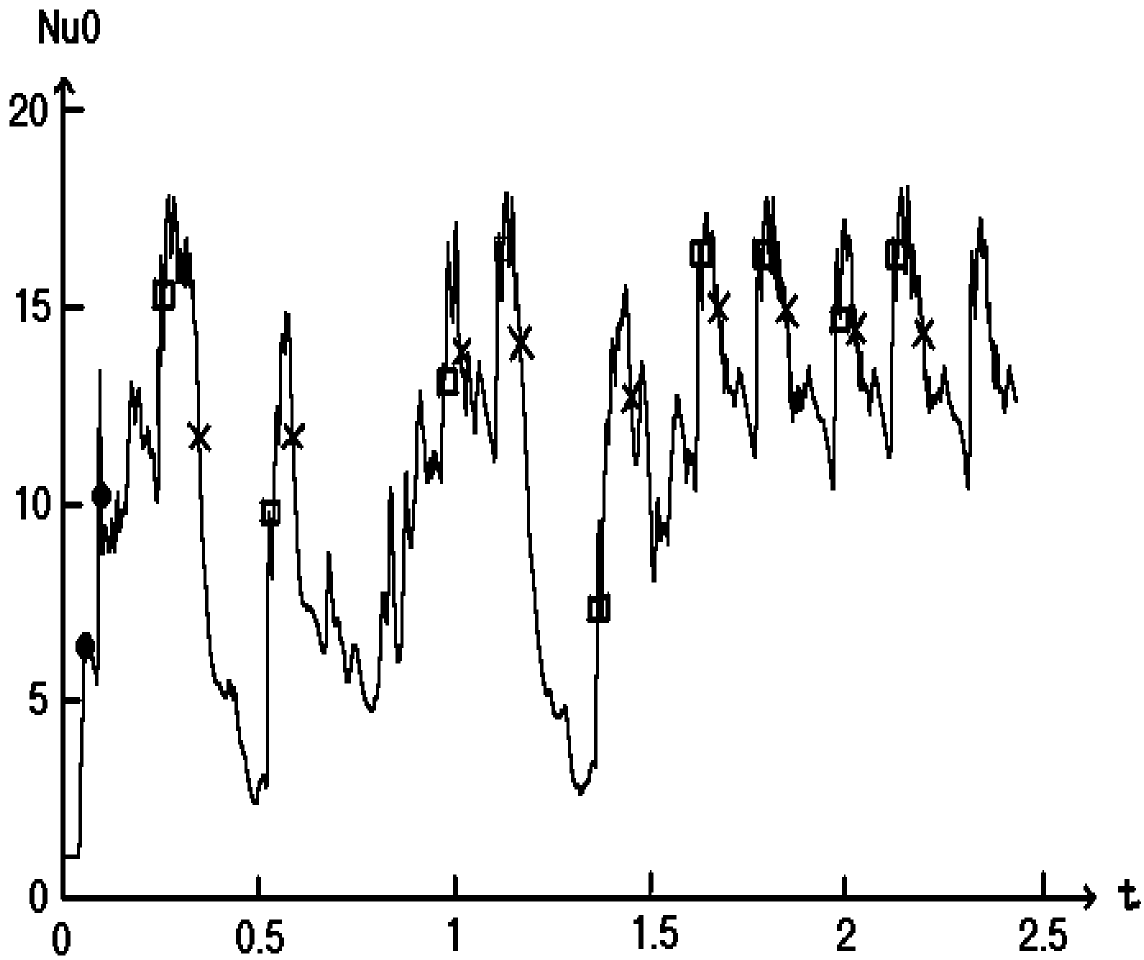
ω(t,x,y) = exp(- ë t) cos(αkx)sin(πmy),
φ(t,x,y) = exp(- ë t) cos(αkx)sin(πmy),
Q(t,x,y) = exp(- ë t) sin(αkx)sin(πmy).



This picture represent of a differential problem (solid line) and numerical method (dashed line) for the first three modes (m=1,2,3). This good coincidence spectral characteristics guarantees exact reproduction infinitesimal of perturbations.

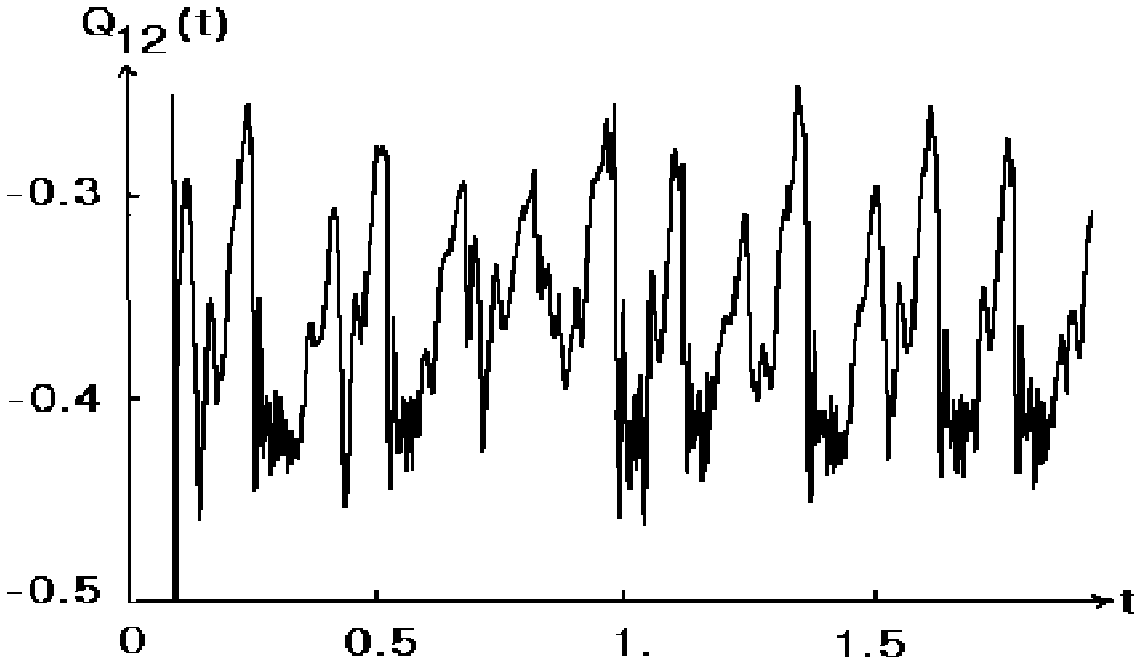
3. Calculation of convectional currents.

We had calculated by this numerical method the stationary (at r 385, then the Hopf bifurcation with birth the periodical regime), periodic (385 r 850, then the rigid bifurcation), quasiperiodic (850 r 890, then the rigid bifurcation into the chaos) and the stochastic (transient) regimes. Results calculation of a stochastic regime convection (r=1000, r=Ra/Racr, Pr=2) here are given.

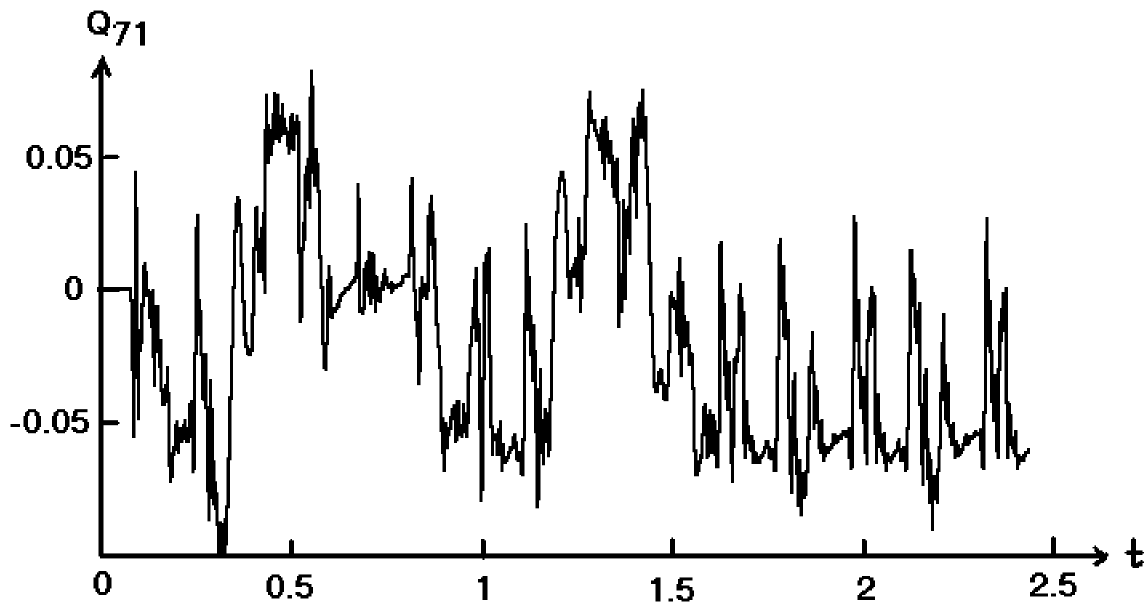


The Nusselt number is represented here as time's function.

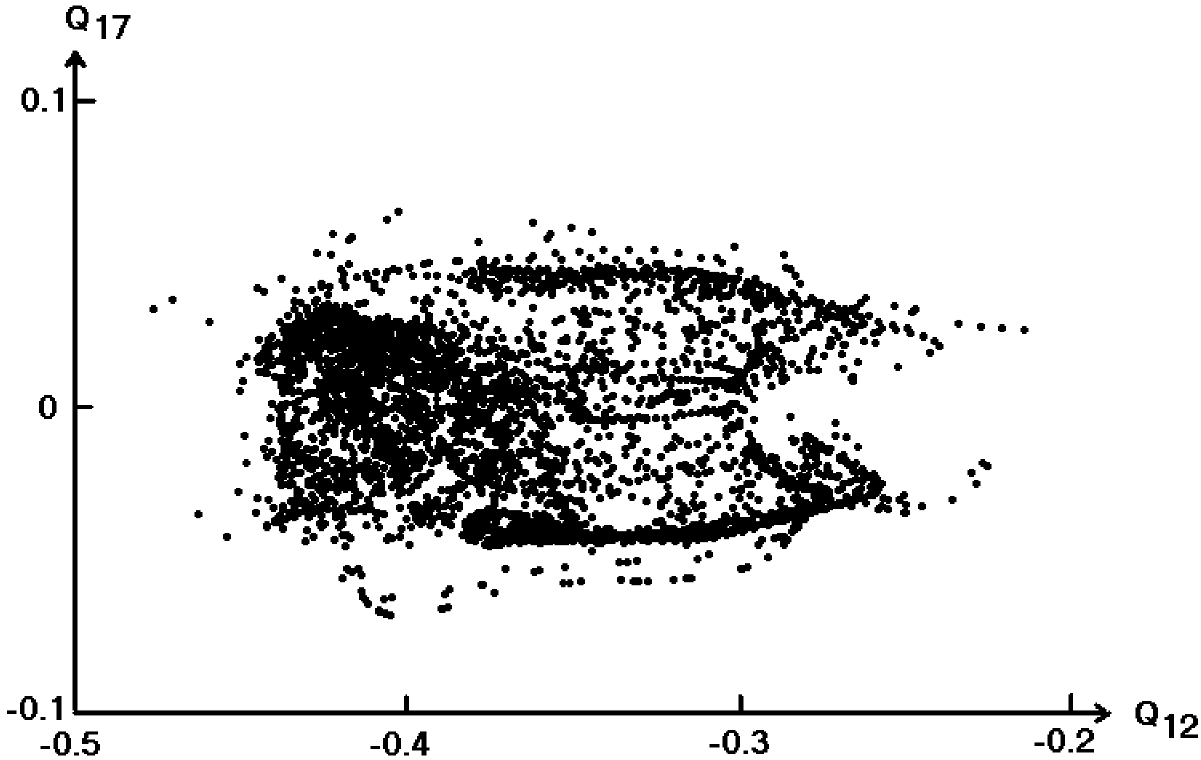
Here, - birth the vortex structure and x – death the vortex structure. The lifetime of this vortex structure is a random quantity with normal law.



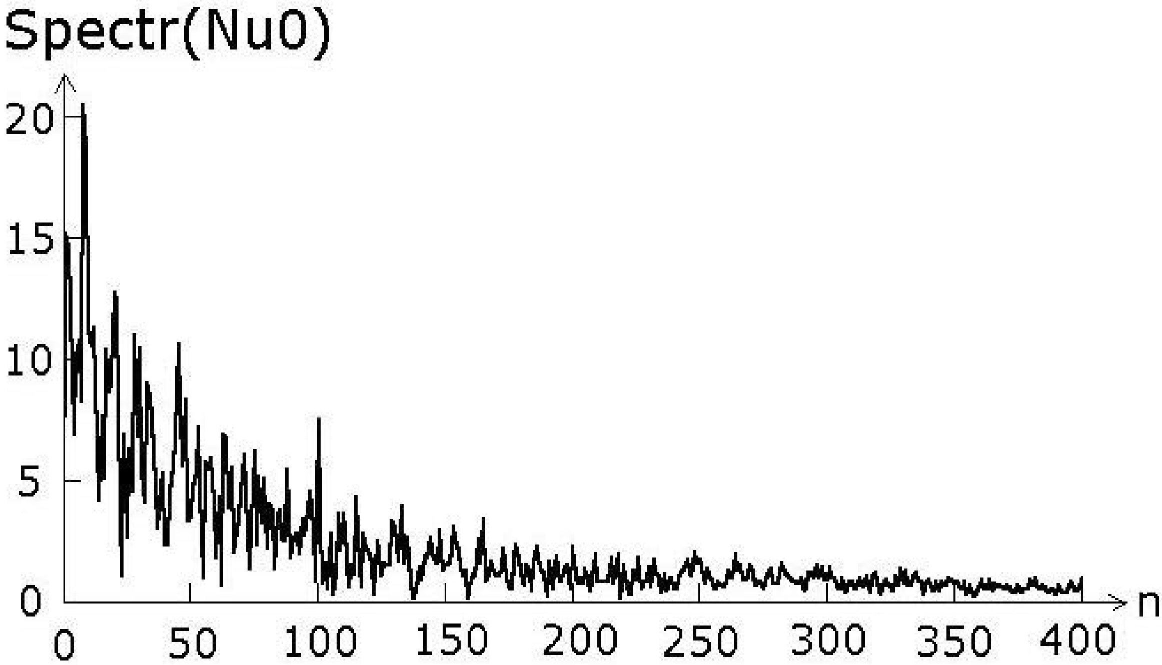
The harmonic Q_{12} (t) is represented as function of time, it is important for exact reflection of energy of flow.



The harmonic Q_{71} (t) has maximal growth in linear approach. It is represent as function of time.



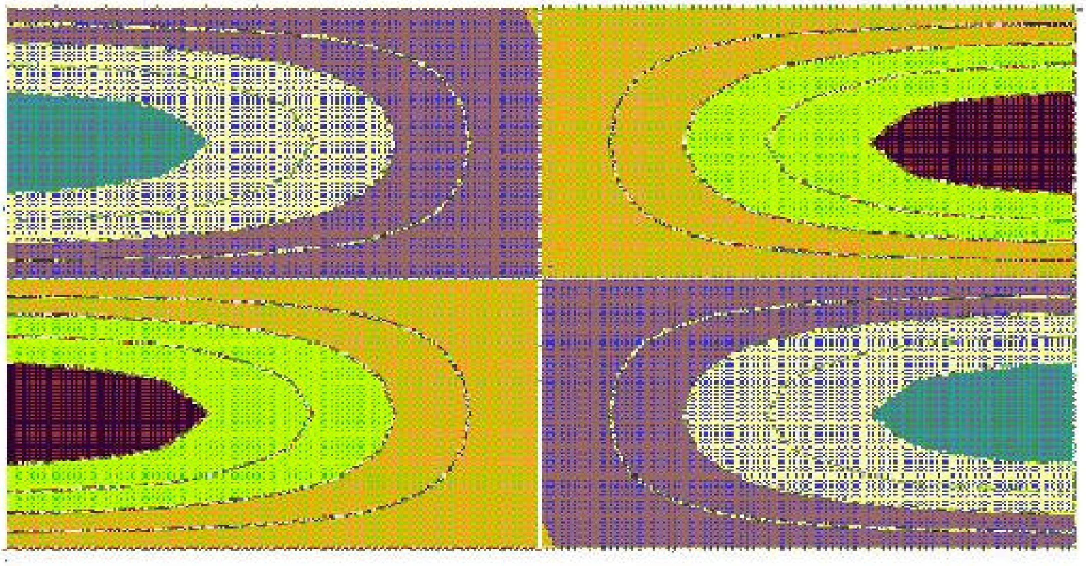
The solution is represent on a phase plane Q_{17}, Q_{12} as function of time what is typical for stochastic process.



Time spectrum of dependence Nu0(t)

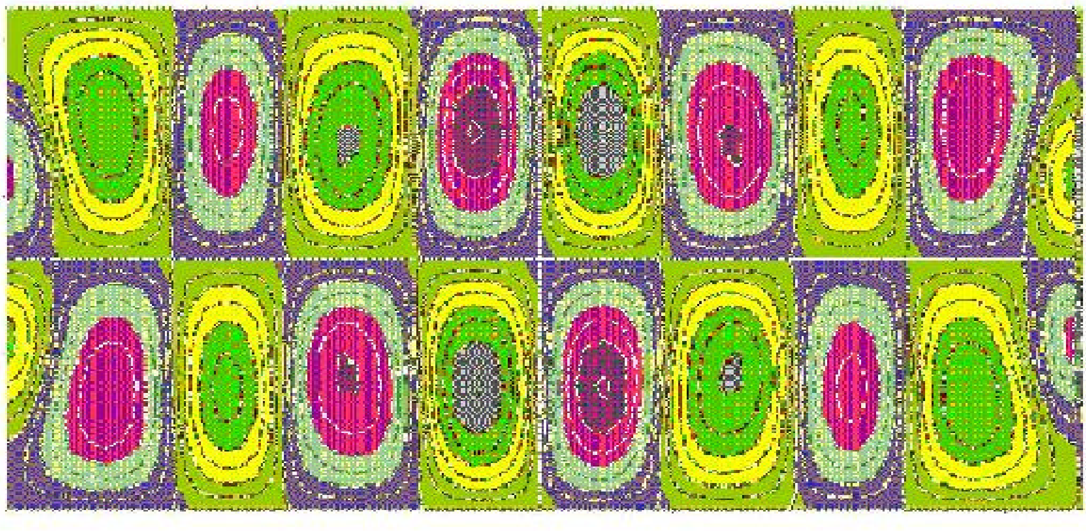
4. Frame of the film.

By results numerical modelling we had generated the film about the stream function development at the time (8.8 MB). Here are single frames this film – the stream function at the different times.



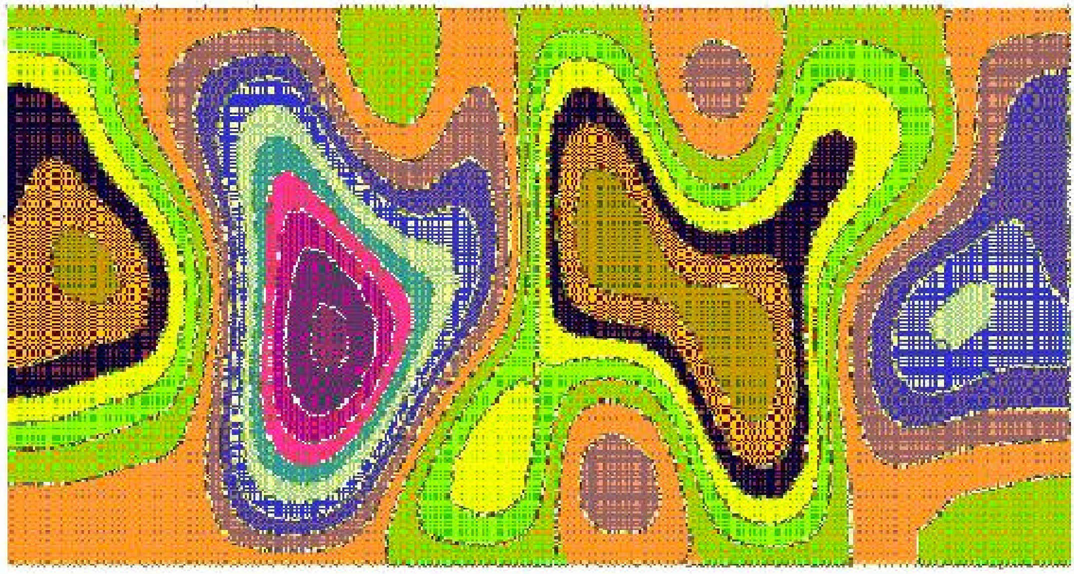
t = 0.01

The initial conditions for the stream function.



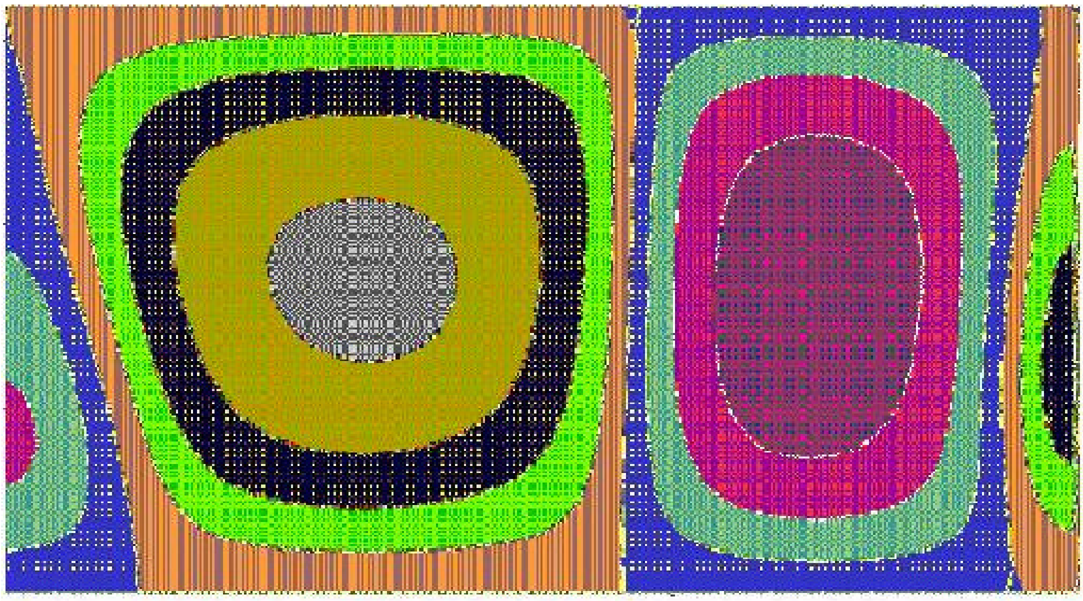
t = 0.059

This is an allocation more unstable mode.



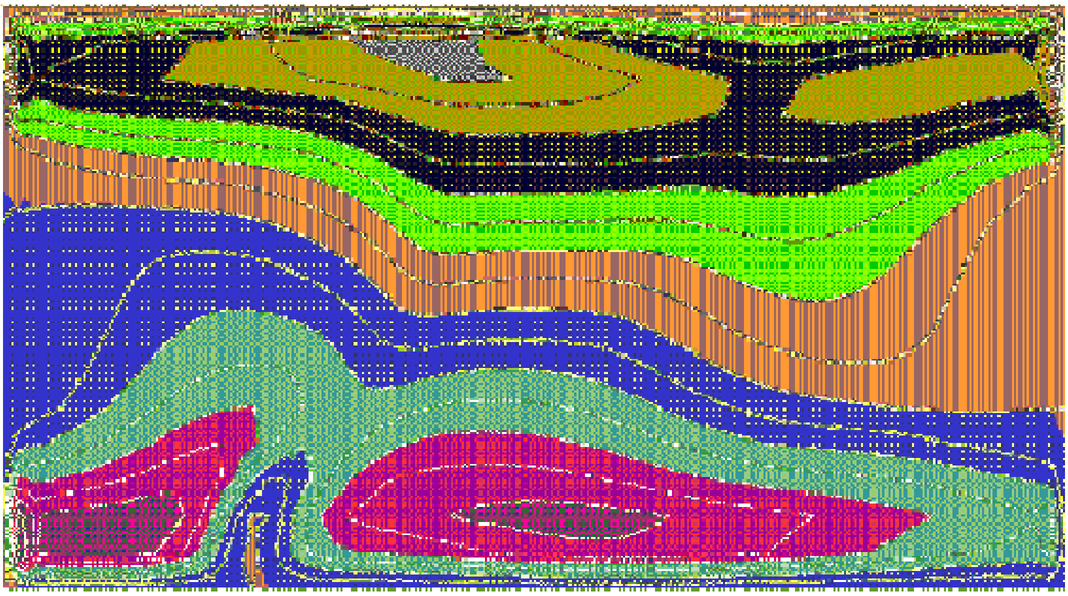
t = 0.087

This is ruin this allocating mode.



t = 0.276

The birth of the vortex structure, then late this vortex structure is death.



t = 0.309

This picture sees the plume come upstairs from below.