DIRECT NUMERICAL SIMULATION OF TURBULENT RAYLEIGH-BENARD CONVECTION

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Introduction

At last time many researchers studied thermal Rayleigh-Benard convection using numerical simulation. As rule, they used spectral methods with periodic boundary conditions. In numerical simulations were derived secondary stationary, periodic, quasiperiodic and stochastic regimes [1-4]. Some authors performed 2-D and 3-D simulations for moderate and high supercriticality with free [5-7,21] and rigid [8,9] boundary conditions on the horizontal plates. The results of correct performed numerical simulations with rigid boundary conditions, as rule, have good agreement with experimental data (see, for example, fig.4 and 5). On the other hand, it seems natural that results of simulations with free and rigid boundary conditions must draw together at enough high supercriticality values. The question of togetherness of solutions with free and rigid boundary conditions is practical, since using of free boundary conditions very simplifies the DNS of turbulent convection, simple and efficient numerical algorithms are generated using the formulas of linear stability theory [10]. It is said in work [6] that difference between solutions with free and rigid boundary conditions is in thin layers near bottom and upper boundaries, since rigid boundaries impose zero values of vertical vorticity on them.

Nevertheless, available data of numerical simulations with free boundary conditions have a bad agreement with experimental data and data of numerical simulations with rigid boundary conditions. In table 1 we compare the data of numerical simulations and experimental data at moderate supercriticality r, here r = Ra/Racr is supercriticality.

Table 1. Comparing of Nusselt number at moderate supercriticality.

r	2-D, free, water[5]	2-D, rigid, water[9]	3-D, rigid, air[8]	exp.in water[11]
750	20.17	8.42	9.01	9.13
1125	23.38	9.47	10.08	10.22

In table 2 we compare the data of numerical simulations by spectral methods in air and water and experimental data in gaseous He at high supercriticality.

Table 2. Comparing of Nusselt number at high supercriticality.

r	[6], free 3-D, air	[7], free 3-D, air	[8], rigid 3-D, air	[9], rigid 2-D, water	[12], experiment in gaseous He
9800	23.0	23.0 ± 0.6	18.3	17.7	18.0
33000	33.0		-	24.8	25.3

In our opinion, these deviations of solutions results are coupled with insufficiently high accuracy of calculations. For correct representation of solution, the exact reproduction of spectral characteristics in linear approach [10] and the enough big number of harmonics [13] are necessary.

The aim of this work is the proof of the togetherness of solutions of convection problems with free and rigid boundary conditions and of experimental data for enough big value of supercriticality.

Problem formulation and numerical method

Turbulent convectional flow in a horizontal layer numerically is simulated at heating from below. The fluid is viscous and incompressible. The flow is time-dependent and two-dimensional. Boundaries of a layer are isothermal and free from shearing stresses. The model Boussinesq is used without semiempirical relationships. The dimensionless system of equations in terms of deviations from an equilibrium solution, representation of problem solution in the form of eigenfunctions sum of linear stability theory, the boundary conditions, the special numerical method and testing are described in work [10]. Solution is periodic, but we consider this solution only in half of period in horizontal direction, therefore the periodic boundary conditions change on the other boundary conditions on the side walls, according to form of solution.

Following a general ideology of the splitting method, transition from layer n to layer n+1 over time is performed in two steps. On the first step, we take into account a linear progress of perturbations neglecting interaction between harmonics. We get the system of two ordinary differential equations for two unknown amplitudes (vortex and temperature deviation) in spectral space solving analytically without any approximations over time. The analytical formulas used here are near to the formulas of linear stability theory. The second step takes into account the nonlinear convection transfer, the interaction between harmonics. Here we use a finite difference method of alternating directions for solving the system equations of nonlinear convective transfer in physical space.

Using of analytical formulas on the first step of splitting guarantees the exact reproduction of spectral curves, it guarantees exact reproduction of infinitesimal perturbations of equilibrium solution.

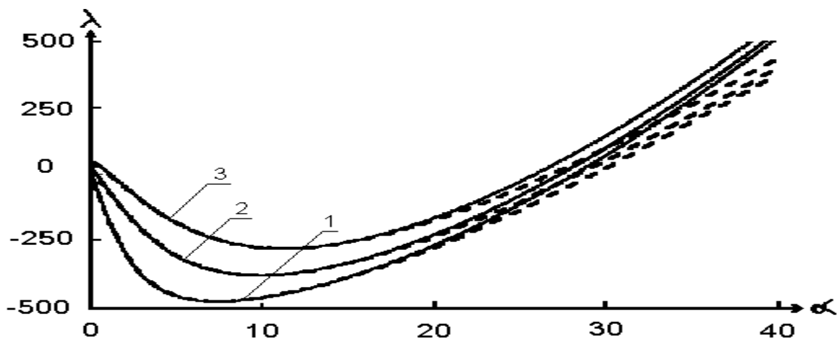

Fig.1. Comparing of the spectral curves.

The fig.1 shows spectral characteristics of differential system (solid line) and of numerical method (dash line) for first three modes at r = 1000, Pr = 2, N = 65, M = 15, here Pr is Prandtl number, N and M are number of harmonics in space directions.

The accuracy of reproduction of finite perturbations we check by performing of integral identity following from equation for temperature.

At little supercriticality (up to $r \approx 50$) the Nusselt numbers from our simulations have a good agreement with results of works [5,14,15].

Establishing of the mean Nusselt number (on the time) determined the duration of simulations on the time.

DNS of turbulent convection

We simulated the convection flows for the Prandtl number Pr = 10, Rayleigh numbers are from 2 up to 16000 times of the critical value. For all simulations the interval of periodicity is equal to 2π. We used 65*15 harmonics for supercriticality r less than 1000 and 129*31harmonics for $r \geq 1000$.

The fig.2 represents the average temperature profile. At the fig.2 and 3 below Y denotes transverse coordinate. At the fig.2 sign ● denotes experimental results [16] (r = 5900, air), dash line – experimental results [11] (r = 5500, water), solid line - results of present work (r = 5800).

The fig.3 represents the root-mean-square of vertical velocity deviation (fluctuations) from average profile one. Here sign ● denotes the experimental results [16] (r = 5900, air), solid line - results of present work (r = 5500).

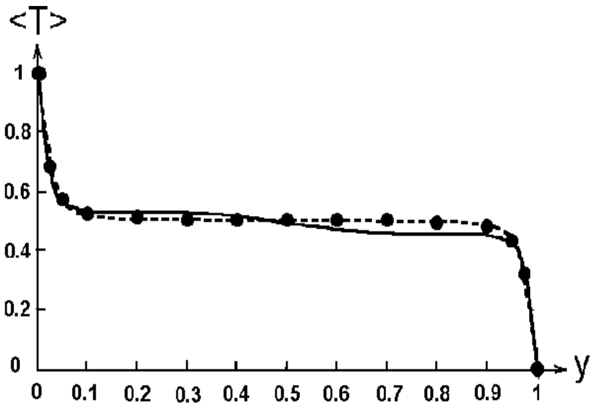

Fig.2. Average temperature profile.

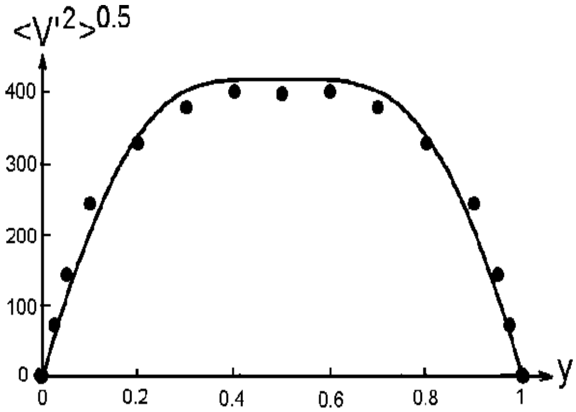

Fig.3. R.m.s. of vertical velocity.

Fig.2 and 3 show that results of numerical 2-D simulation are consistent with experimental data at supercriticality $r \approx 6000$.

The r.m.s. values of temperature fluctuations in the center (at Y = 0.5) are plotted on the fig. 4 versus supercriticality r up to 16000. Here ● is numerical result of present work (Pr = 10), dash line – 3-D numerical simulation [8] (air), dadot – experimental results [19] (air), sing X – denotes experimental results [20] (only single point, air), sing ○ - experimental results [16] (air), sing □ - 3-D numerical simulation with free boundary conditions [21].

The r.m.s. values of vertical velocity fluctuations in the centre (at Y = 0.5) are plotted on the fig. 5 versus supercriticality r up to 16000. Here ● is numerical result of present work (Pr = 10), dash line – 3-D numerical simulation [8] (air), dadot – experimental results [19] (air), sing □ - 3-D numerical simulation with free boundary conditions [21].

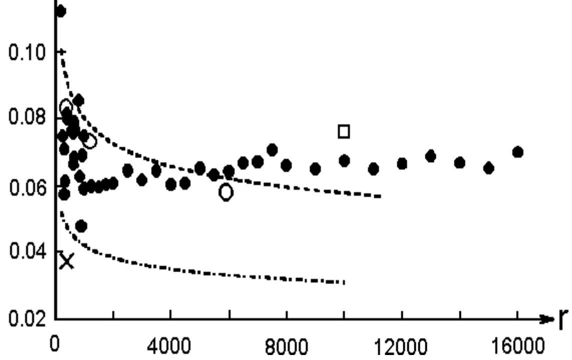

Fig.4. R.m.s. of temperature fluctuations.

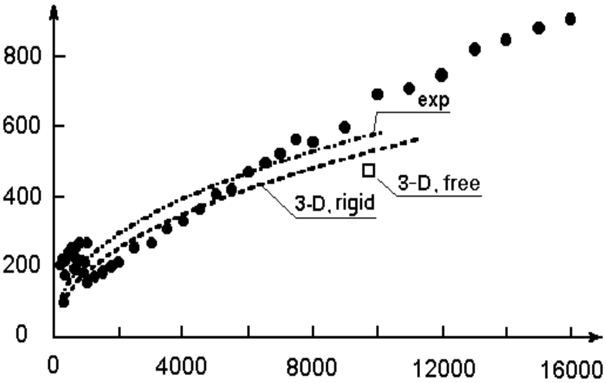

Fig. 5. R.m.s. of vertical velocity fluctuations.

Fig. 4 and 5 show that data of our 2-D numerical simulation is consistent with experimental data and results of 3-D numerical simulation, a big scatter of numerical data up to $r \approx 1000$ mirrors the existent scatter in experimental data (fig.4 and work [16]).

The fig. 6 represents mean (time-average) Nusselt number versus supercriticality from 200 up to 16000. Here sign ● denotes the results of present work, solid line - experimental results [11] (water), □ - 3-D numerical simulation [8] (air), dadot line - the results of the experimental work [12] (helium gas), sign ○ - 2-D simulation [9] (water) and sign ■ denotes the numerical results of works [6,7] with free boundary conditions.

A least squares fit for Nusselt number versus r from our simulations is

$$Nu = 1.884 r^{0.243},$$

this formula is right for $r \geq 800$. It is interesting, that such low values of exponent are more typical for fluids with low Prandtl numbers, for example, for mercury [18]. Possibly, it is coupled with restrictions of 2-D simulation.

At fig.7 also represents mean Nusselt number versus supercriticality, but for $20 \leq r \leq 1500$. Here sign ● - results of present work, □ - numerical simulation [8], ◊ - numerical simulation [17], x - numerical simulation [9], dash line - numerical simulation [5] with free boundary conditions, matastable part of regime 1 (for $340 \leq r \leq 610$) and experiment [18] (water, fat line), solid line – experiment [11] (water) and sing ○ - experiment Silveston, 1958 (water, these results from work [18]).

Table 3. Comparing of Nusselt Number for free and rigid boundary conditions.

r	Present (2-D, free)	[9] (2-D, rigid)	[8] (3-D, rigid)
750	9.76	8.42	9.02
1500	11.22	10.41	10.92
3000	13.19	12.86	13.22
6000	15.65	15.59	16.01
12000	18.56	18.79	19.39

Data of work [8] were calculated on the formula $Nu = 1.45 \cdot r^{0.276}$ from this work. The results of 2-D simulations (present work and work [9]) are practically coincide for $r \geq 3000$, the results of present work and results of 3-D simulation [8] have reasonably good agreement.

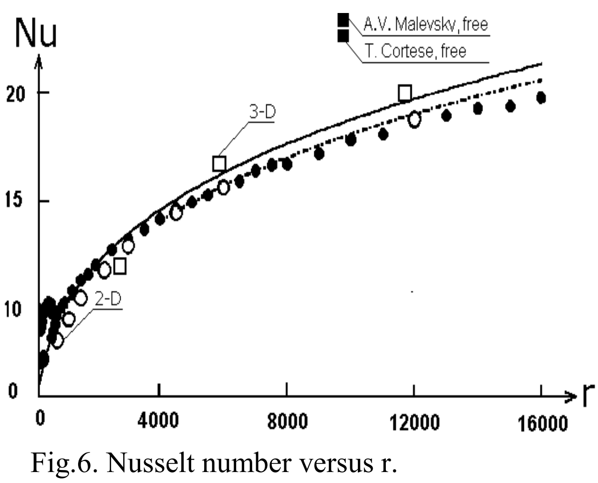

Fig.6. Nusselt number versus r.

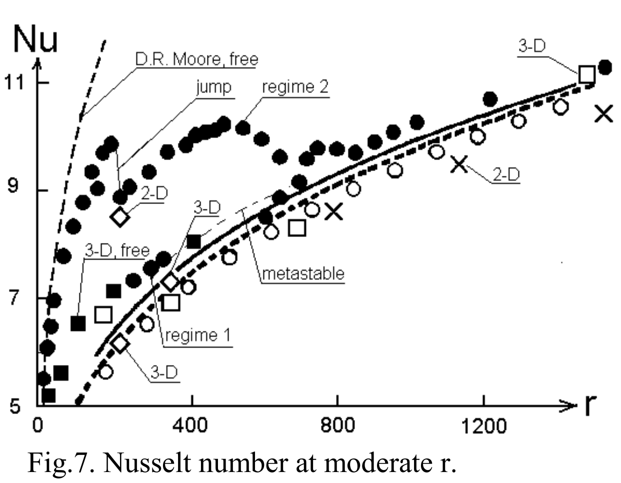

Fig.7. Nusselt number at moderate r.

We note that for $r \leq 700$ we derived numerically two different regimes of convection. Nusselt numbers of regime1 have reasonably good agreement with both experimental results and results of numerical simulations with rigid boundary conditions. For $340 \leq r \leq 610$ regime 1 is metastable in 2-D simulations. For $r \approx 200$ in regime 2 is neatly seen a jump in dependence Nusselt number versus r, also in experimental work [12] for $r \approx 200$ was discovered a same jump in heat flux and some hysteresis.

Conclusion

In conclusion, we emphasize that results of our 2-D simulations with free boundary conditions on the horizontal plates are closely allied both with results of 2-D and 3-D simulations with rigid boundary conditions on the horizontal plates and experimental data for enough high values of supercriticality. In particular, the values of Nusselt number at $r > 800$ describe by formula:

$$Nu = 1.884 r^{0.243},$$

it is close to experimental data and data of numerical simulations with rigid boundary conditions. The profiles of mean temperature and r.m.s. of vertical velocity fluctuations are typical for stochastic regimes and turbulent convection. For $r \approx 6000$ these profiles and experimental data are close. Some loss of simulation accuracy for high supercriticality ($r > 10^4$, fig.5) possibly is coupled with restrictions of 2-D simulation. The scatter of numerical data (up to $r \approx 1000$) and existence of various regimes of convection (up to $r \approx 700$) mirror the existent scatter in experimental data.

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