Paper 12393

DISORDER OF THE FRONT OF A TENSILE TUNNEL-CRACK PROPAGATING IN SOME INHOMOGENEOUS MEDIUM

Jean-Baptiste Leblond

Laboratoire de Modélisation en Mécanique,
Université Pierre et Marie Curie, Paris, France

A number of theoretical, numerical and experimental works have recently been devoted to the study of geometrical disorder of the front of planar cracks propagating in inhomogeneous solids, with applications to propagation of cracks in composites, and geological faults during earthquakes. Prominent contributions in this field are notably due to Rice and coworkers, who considered the case of a semi-infinite crack propagating dynamically in an infinite brittle material with random toughness. They showed that the "degree of disorder" of the crack front, as measured by the autocorrelation function of the distance from the actual front to some "reference" straight front, increases in time without bound.

No characteristic lengthscale was included in the geometry considered by Rice and coworkers. The aim of this paper is to introduce this feature in such theoretical studies. This is done by considering the simplest model crack geometry involving some characteristic length, namely some tunnel-crack of finite width in some infinite medium.

Although the geometry considered is still very simple, the problem becomes notably more involved and it reveals impossible to treat it with the same hypotheses as Rice and coworkers; simplifications are compulsory. These simplifications are two-fold : dynamic effects are disregarded and the Griffith crack propagation law, appropriate for brittle solids, is replaced by some Paris law. This law involves 2 material parameters, some constant C and some exponent N ; it is adequate for fatigue or subcritical crack growth and "simulates" Griffith's law in some sense in the limit of very large Paris exponent N . Also, to simplify the treatment, it is assumed that the crack is loaded in pure mode I, that its front remains symmetrical with respect to its middle line at all instants, and that the sole Paris constant C fluctuates within the material, the exponent N being uniform.

The treatment uses a first-order perturbative approach which assumes that the perturbation $\delta a(z)$, that is the local distance (at position z) from the actual crack front to some "reference" straight front, remains small as compared to the mean half-width a of the crack at all instants. A formula providing the variation of the stress intensity factor along the crack front in terms of its perturbation $\delta a(z)$ was derived a few years ago by Leblond, Mouchrif and Perrin. Using this formula, a Fourier transform of the perturbation in the direction z of the crack front, and the propagation law, one derives evolution equations for all Fourier components of the perturbation. Since the analysis is performed within a linearized context, the evolutions of the various Fourier components are found to be independent of each other. The evolution equations can be integrated in time explicitly so as to yield the expression of each Fourier component at any instant in terms of its initial value, thus opening the way to study of the development of geometrical disorder of the front.

This disorder is evaluated via the autocorrelation function of the perturbation $\delta a(z)$. This autocorrelation function can be expressed, at all instants, in terms of the autocorrelation function of the Paris constant C. The formula obtained allows for study of the asymptotic expression, for large times, of various quantities related to the autocorrelation function (power spectrum of the perturbation, root mean square deviation of the front from a straight line, etc.).

It is found that the disorder of the front increases without bound for large times, at a considerable rate; for instance, the root mean square deviation from a straight line increases proportionally to $(a/a_0)^{(N-1)/2}$ where a_0 denotes the initial value of the mean half-width a of the crack. However, this conclusion is mitigated by the fact that the “correlation distance” of the crack front, that is the characteristic distance over which the advance at some point “influences” the advance at other points, also increases, proportionally to a. This suggests that the crack front tends to “straighten” in time in some sense.

The growth rate of disorder found in this work is much larger than that found by Rice and coworkers. This is necessarily due to one of the differences between the hypotheses adopted in their works and here, namely consideration, in the present work, of a crack with finite width, neglect of dynamic effects, and use of Paris’s propagation law instead of Griffith’s. But it can be shown that the last 2 differences can only generate a lower growth rate of disorder in the present work. Therefore the higher growth rate actually found can only arise from the first difference, that is existence of some characteristic length in the problem envisaged here.

The “destabilizing” effect of the finite geometry considered can be qualitatively explained as follows. The theoretical analysis reveals that perturbations with wavelength greater than some critical value grow in time, whereas those with wavelength smaller than it decay; the “critical wavelength” is proportional to the mean half-width of the crack. Thus the finite crack geometry envisaged is intrinsically “unstable” versus perturbations of large wavelength. This effect disappears for a semi-infinite crack, because the critical wavelength is infinite for such a geometry, so that perturbations of all wavelengths decay.

Although dynamic effects could not be included for technical reasons in the present work, it seems reasonable to infer from it that existence of some characteristic length should also enhance the growth rate of disorder of the front of a crack propagating dynamically. The implication for the study of propagation of geological faults during earthquakes is that it would be highly desirable to consider finite cracks rather than semi-infinite ones in future studies, in spite of the technical difficulties implied.