

# CONTACT ZONE APPROACH TO THE ANALYSIS OF INTERFACE CRACKS IN THERMOMECHANICALLY LOADED PIEZOELECTRIC BIMATERIALS

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**Summary** An interface crack with a mechanically frictionless contact zone in a piezoelectric bimaterial under the action of a remote mixed mode mechanical loading as well as of thermal and electrical fields is considered. Electrically permeable and electrically insulated conditions at the open part of the crack are taken into consideration. The influence of the mechanical loading as well as of the thermal and electrical fluxes upon the electromechanical values is studied.

The interface crack investigation in a piezoelectric bimaterial, because of its importance, attracts an essential attention during the last years. The most investigations on this subject are performed by means of the “open” crack model possessing an oscillating singularity at the crack tips (Shen and Kuang [1], Gao and Wang [2]). Only in the paper by Qin and Mai [3] a thermopiezoelectric bimaterial with an electrically impermeable interface crack under the assumption of a contact zone model has been investigated by means of the method of singular integral equations, and moreover, electro-mechanically loaded electrically permeable and impermeable interface cracks having a contact zone in a piezoelectric bimaterial have been analytically studied by Herrmann and Loboda [4] and Herrmann et al. [5], respectively.

But the problem of the correct choose of the electrical conditions at the interface crack faces remains still insufficiently clear, and therefore, many investigations are concerned with an electrically permeable interface crack whilst in another group of investigations an electrically impermeable interface crack is considered.

In this study due to an exact analytical investigation of an interface crack with a contact zone in a piezoelectric bimaterial space the influence of the mechanical, thermal and electrical fields upon the fracture mechanical characteristics is analyzed for different electrical conditions at the crack faces.

Consider a crack situated in the region  $c \leq x_1 \leq b$ ,  $x_3=0$  between two semi-infinite spaces ( $x_3 > 0$ , material 1 and  $x_3 < 0$ , material 2) which are loaded at infinity with uniform stresses  $\sigma_{33}^{(m)} = \sigma$ ,  $\sigma_{13}^{(m)} = \tau$  and  $\sigma_{11}^{(m)} = \sigma_{xm}^{\infty}$  as well as with uniform electric fluxes  $D_3^{(m)} = d$ ,  $D_1^{(m)} = D_{xm}^{\infty}$  satisfying the continuity conditions at the interface. Besides, a uniform temperature flux  $q_0$  in the  $x_3$ -direction is imposed at infinity. It is assumed that the crack surfaces are traction-free for  $x_1 \in (c, a)$  whilst they are in frictionless contact for  $x_1 \in (a, b)$ , and the position of the point  $a < b$  is arbitrarily chosen for the time being. It means that the interface conditions for the thermally perturbed state have the following form

$$x_1 \notin (c, b) : [\mathbf{V}(x_1, 0)] = 0, [\mathbf{t}(x_1, 0)] = 0, \quad (1)$$

$$x_1 \in (c, a) : q_3^{\pm} = -q_0, \sigma_{13}^{(m)}(x_1, 0) = 0, \sigma_{33}^{(m)}(x_1, 0) = 0, [D_3(x_1, 0)] = 0, [\varphi(x_1, 0)] = 0 \quad (2)$$

-for an electrically permeable crack,

$$q_3^{\pm} = -q_0, \mathbf{t}^{(m)}(x_1, 0) = 0 \text{ - for an electrically impermeable crack,} \quad (3)$$

$$x_1 \in (a, b) : [T] = 0, [q_3] = 0, [u_3(x_1, 0)] = 0, \sigma_{13}^{(m)}(x_1, 0) = 0, [\sigma_{33}(x_1, 0)] = 0, [D_3(x_1, 0)] = 0, [\varphi(x_1, 0)] = 0. \quad (4)$$

where  $[\mathbf{V}'(x_1, 0)] = \mathbf{V}'^{(1)}(x_1, 0) - \mathbf{V}'^{(2)}(x_1, 0)$ ,  $\mathbf{V} = [u_1, u_2, u_3, \varphi]^T$ ,  $\mathbf{t} = [\sigma_{13}, \sigma_{23}, \sigma_{33}, D_3]^T$ .

Assuming all fields are independent on the coordinate  $x_2$  for the solution of the formulated problem the expressions of the temperature jump  $[T(x_1)]$  across the material interface, the temperature flux  $q_3$  as well as the vectors  $[\mathbf{V}']$  and  $\mathbf{t}$  via the sectionally-holomorphic functions are constructed similarly to the presentations of Shen and Kuang [1]. Due to these presentations the thermal problem can be reduced to a relatively simple Hilbert problem and solved exactly both for an electrically permeable and an electrically impermeable interface crack. Moreover, the satisfaction of the electromechanical boundary conditions (1)-(4) leads for an electrically permeable interface crack to the following inhomogeneous combined Dirichlet-Riemann problem for a sectionally-holomorphic function  $F(z)$

$$F^+(x_1) + \gamma F^-(x_1) = \Psi_1(x_1) \text{ for } x_1 \in (c, a) \quad (5)$$

$$\text{Im } F^{\pm}(x_1) = \Psi_2(x_1) \text{ for } x_1 \in (a, b), \quad (6)$$

where the functions  $\Psi_1(x_1)$  and  $\Psi_2(x_1)$  are defined by the thermal solution obtained above. It is important that the solution of the problem of the linear relationship (5), (6) has been presented in a closed form similarly to those given by Herrmann and Loboda [6].

For an electrically impermeable interface crack a Hilbert problem appears in addition to the problem (5), (6), but nevertheless the analytical solutions of all obtained problems are found and all necessary thermal, mechanical and electrical characteristics at the interface are presented in a closed form. Moreover, the closed analytical formulas for the stress intensity factors (SIFs)

$$k_1 = \lim_{x_1 \rightarrow a+0} \sqrt{2\pi(x_1 - a)} \sigma_{33}^{(1)}(x_1, 0), \quad k_2 = \lim_{x_1 \rightarrow b+0} \sqrt{2\pi(x_1 - b)} \sigma_{13}^{(1)}(x_1, 0)$$

as well as the electrical displacement intensity factor  $k_4 = \lim_{x_1 \rightarrow a+0} \sqrt{2\pi(x_1 - a)} D_3^{(1)}(x_1, 0)$  have been obtained for an

arbitrary value of the relative contact zone length  $\lambda = \frac{b-a}{b-c}$ .

Further, the real contact zone length could be obtained. For this purpose additional conditions  $\sigma_{33}^{(1)}(x_1, 0) \leq 0$  for  $x_1 \in (a, b)$ ,  $[u_3(x, 0)] \geq 0$  for  $x_1 \in (c, a)$  should be satisfied. For an electrically permeable crack the satisfaction of these inequalities gives a transcendental equation with respect to the relative contact zone length  $\lambda$ . The largest root  $\lambda_0$  of this equation from the interval (0, 1) defines the required real contact zone length. This root can be found numerically, but for a small  $\lambda_0$  an analytical formula has been obtained as well.

The numerical results were obtained for a bimaterial composed of piezoelectric Cadmium Selenium (Ashida and Taushert [7]) (the upper material) and glass (the lower one). The variation of the relative contact zone length  $\lambda_0$  and the correspondent SIF  $k_2$  with respect to the intensity of the thermal flux  $q_0$  and the coefficients of the normal-shear loading  $k = \tau/\sigma$  are studied. It is worth to note that for an electrically permeable interface crack neither the contact zone length nor the intensity factors depend on the intensity of the electrical flux  $d$ .

A more complicated situation concerning the determination of the real contact zone length takes place for an electrically impermeable interface crack. In this case the inequalities mentioned above hold true if  $a$  is taken from the segment  $[a_1, a_2]$  providing  $a_1 \leq a_2$  holds true, where  $a = b - \lambda l$ ,  $a_1 = b - \lambda_1 l$ ,  $a_2 = b - \lambda_2 l$  and  $\lambda_1$  is the maximum root from the interval (0, 1) of the equation  $k_1 = 0$  and  $\lambda_2$  is the similar root of the equation

$\lim_{x_1 \rightarrow a-0} \sqrt{a - x_1} [u_3'(x_1, 0)] = 0$ . The required roots of the mentioned equations can be found numerically or analytically

(for small  $\lambda_1$  and  $\lambda_2$ , respectively), and the segment  $[a_1, a_2]$  can be obtained. An additional analysis based upon the theorem of the minimum potential energy shows that the real position of the point  $a$  coincides with  $a_1$  providing  $a_1 \leq a_2$  holds true. If the last inequality is not valid then the inequalities from above cannot be satisfied and other thermal and electrical interface conditions should be introduced.

The numerical analysis showed that for the electrical flux  $d=0$  the contact zone lengths correspondent to an electrically insulated or an electrically permeable crack are practically the same. However, a nonzero electrical flux changes the real contact zone length and the associated fracture mechanical parameters for an electrically impermeable interface crack. The essential dependencies of these values on  $d$  appear for rather large intensities of the electrical flux only. This is particularly demonstrated by the Table 1 where the values of the relative contact zone length  $\lambda_1$  are shown with respect to the intensity of the electrical flux  $d^* = c_{33}^{(1)} d / (e_{33}^{(1)} \sigma)$  for  $q_0=0$ ,  $k=0$  and for the same bimaterial as earlier.

Table 1

$d^*$	$10^3$	$10^4$	$2 \times 10^4$
$\lambda_1$	$6.38 \times 10^{-13}$	$6.62 \times 10^{-4}$	$0.0177$

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