ON 3-D THERMOELASTIC PROBLEMS OF INTERFACIAL CRACKS IN A PERIODIC STRATIFIED SPACE

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Summary The methods of solutions to three-dimensional stationary thermoelastic problems involving a two-layered microperiodic space containing an interface crack of arbitrary smooth profile. An approximate analysis is performed within the framework of linear stationary thermoelasticity with microlocal parameters. The resulting boundary-value problems are reduced to the corresponding ordinary crack problems with mechanical loading in homogeneous isothermal elasticity.

INTRODUCTION

The rapidly increasing use of new composite materials with a large number of dissimilar layers in advanced engineering structures requires the study of different aspects of their fracture behavior. A great deal attention has been drawn to interfacial cracking. It is well known that many conventional solutions for interfacial cracks have oscillatory singularities, which causes the overlapping of crack faces. To eliminate these unsatisfactory features, several interfacial zone models have been proposed and discussed (see e.g. a review in [1]).
In this paper, an approximate theory called the linear stationary thermoelasticity with microlocal parameters, devised in [2,3], is used to analyze three-dimensional problems with a crack of arbitrary shape lying on an interface in a periodic two-layered elastic space. The advantage of this approach is a relatively simple form of the governing equations appearing similar to the thermoelasticity for transverse isotropy, which makes it possible to construct the appropriate potentials and establish an analogy between the thermal crack problems and their mechanical counterparts. Thus, the typical inverse square root crack-border stress singularities, characterized by the stress intensity factors, are obtained.

PROBLEM FORMULATION AND GOVERNING RELATIONS

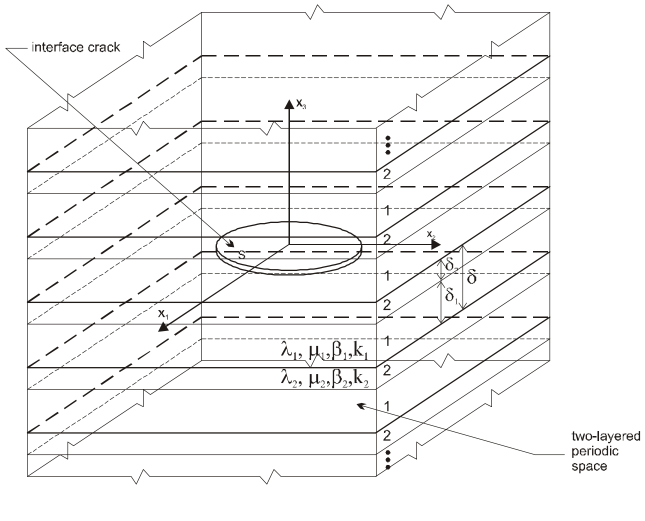

Fig. Two-layered periodic space with an interface crack

Consider a bimaterial-laminated space as shown in Figure. A repeated fundamental layer of thickness δ is composed of two homogeneous sublayers, denoted by $l=1$ and $l=2$, with thicknesses δ_1 and δ_2, and characterized by different thermo-mechanical properties. Let λ_l, μ_l be the Lamé constants, k_l the thermal conductivities, $\beta_l/\left(\lambda_l+\frac{2}{3}\mu_l\right)$ the coefficients of the volume expansion, $l=1,2$.
To determine the temperature-stress fields in this laminated body, the linear stationary thermoelasticity with microlocal parameters ([2], [3]) is applied to seek an approximate solution. Without going into details we recall below only the final governing equations and constitutive relations of the macro-homogeneous homogenized model of the treated body (see [4] for the details of the derivation; summation over repeated subscript $\gamma = 1{,}2$ is taken for granted):

– the equation of heat conduction for a macro-temperature ϑ (in the absence of heat sources)

$$\vartheta_{,11}+\vartheta_{,22}+k_0^{-2}\vartheta_{,33}=0 \tag{1}$$

– the equations for macro-displacements w_i (in the absence of the body forces)

$$\begin{gathered}0{,}5\left(c_{11}+c_{12}\right)w_{\gamma,\gamma\alpha}+0{,}5\left(c_{11}-c_{12}\right)w_{\alpha,\gamma\gamma}+\\+c_{44}w_{\alpha,33}+\left(c_{13}+c_{44}\right)w_{3,3\alpha}=K_1\vartheta_{,\alpha}, \quad \alpha=1,2,\\\left(c_{13}+c_{44}\right)w_{\gamma,\gamma3}+c_{44}w_{3,\gamma\gamma}+c_{33}w_{3,33}=K_3\vartheta_{,3}\end{gathered} \tag{2}$$

– the constitutive relations for the fluxes $q_i^{(l)}$ and the stresses $\sigma_{ij}^{(l)}$ in the layer of the l th kind

$$\begin{aligned}&q_\alpha^{(l)}=-k_l\vartheta_{,\alpha}, \quad \alpha=1,2, \qquad q_3^{(l)}=-K\vartheta_{,3},\\&\sigma_{\alpha3}^{(l)}=c_{44}\left(w_{\alpha,3}+w_{3,\alpha}\right), \quad \alpha=1,2,\\&\sigma_{33}^{(l)}=c_{13}\left(w_{1,1}+w_{2,2}\right)+c_{33}w_{3,3}-K_3\vartheta,\\&\sigma_{12}^{(l)}=\mu_l\left(w_{1,2}+w_{2,1}\right),\\&\sigma_{11}^{(l)}=d_{11}^{(l)}w_{1,1}+d_{12}^{(l)}w_{2,2}+d_{13}^{(l)}w_{3,3}-K_2^{(l)}\vartheta,\\&\sigma_{22}^{(l)}=d_{12}^{(l)}w_{1,1}+d_{11}^{(l)}w_{2,2}+d_{13}^{(l)}w_{3,3}-K_2^{(l)}\vartheta.\end{aligned} \tag{3}$$

Positive constants appearing in the above equations, describing material and geometrical characteristics of the composite constituents are given in the sited paper [4]. Observe that the condition of a perfect bond between the layers is satisfied, and setting $\mu_1 = \mu_2 \equiv \mu$, $\lambda_1 = \lambda_2 \equiv \lambda$ and $\beta_1 = \beta_2 \equiv \beta$, $k_1 = k_2 \equiv k$ we get $c_{11} = c_{33} = \lambda + 2\mu$, $c_{12} = c_{13} = \lambda$, $c_{44} = \mu$, $K_1 = K_3 = \beta$, $K = k$, $k_0 = 1$, passing directly to the well-known equations of stationary thermoelasticity for a homogeneous isotropic body.

The solution of the governing equations (2) is dependent on the material constants of the sublayers. In the general case $\mu_1 \neq \mu_2$, $t_\alpha \neq k_0$ (the other cases are detailed in [4]; all constants appearing are given in [4]) it is expressed in terms of three harmonic potentials $\phi_i(x_1, x_2, z_i)$, $z_i = t_i x_3$, $i = 1,2,3$ and the temperature harmonic potential $\omega(x_1, x_2, z_0)$, $z_0 = k_0 x_3$, related to the solution of (1) such that $\vartheta(x_1, x_2, x_3) = -\dfrac{\partial^2 \omega(x_1, x_2, z_0)}{\partial^2 z_0}$, as follows

$$
\begin{aligned}
w_1 &= \left(\phi_1 + \phi_2 + c_1\,\omega\right)_{,1} - \phi_{3,2}\,, \\
w_2 &= \left(\phi_1 + \phi_2 + c_1\,\omega\right)_{,2} + \phi_{3,1}\,, \\
w_3 &= \sum_{\alpha=1}^{2} m_\alpha\, t_\alpha \frac{\partial \phi_\alpha}{\partial z_\alpha} - c_2\, k_0 \frac{\partial \omega}{\partial z_0}\,,
\end{aligned}
\qquad
\begin{aligned}
\sigma_{31} &= c_{44}\left[\sum_{\alpha=1}^{2}(1+m_\alpha)t_\alpha \frac{\partial \phi_\alpha}{\partial z_\alpha} + (c_1 - c_2)k_0 \frac{\partial \omega}{\partial z_0}\right]_{,1} - t_3 \frac{\partial^2 \phi_3}{\partial z_3 \partial x_2}\,, \\
\sigma_{32} &= c_{44}\left[\sum_{\alpha=1}^{2}(1+m_\alpha)t_\alpha \frac{\partial \phi_\alpha}{\partial z_\alpha} + (c_1 - c_2)k_0 \frac{\partial \omega}{\partial z_0}\right]_{,2} + t_3 \frac{\partial^2 \phi_3}{\partial z_3 \partial x_1}\,, \\
\sigma_{33} &= c_{44}\left[\sum_{\alpha=1}^{2}(1+m_\alpha) \frac{\partial \phi_\alpha}{\partial z_\alpha} + \alpha_0 \frac{\partial^2 \omega}{\partial z_0^2}\right].
\end{aligned}
\tag{4}
$$

SOLUTIONS TO THERMALLY LOADED INTERFACE CRACKS

Of interest are two general problems (symmetric, denoted by 1 and skew-symmetric, denoted by 2) related to the half-plane $x_3 \geq 0$ when the boundary plane $x_3 = 0$ is subjected to the following general thermal-stress conditions:

Problem 1:

$$
\begin{aligned}
&\vartheta(x_1, x_2, 0^+) = -T_0(x_1, x_2), \quad (x_1, x_2) \in S \\
&\vartheta_{,3}(x_1, x_2, 0^+) = 0, \quad (x_1, x_2) \in Z - S, \\
&\sigma_{31}(x_1, x_2, 0^+) = \sigma_{32}(x_1, x_2, 0^+) = 0, \quad (x_1, x_2) \in Z, \\
&\sigma_{33}(x_1, x_2, 0^+) = 0, \quad (x_1, x_2) \in S, \\
&w_3(x_1, x_2, 0^+) = 0, \quad (x_1, x_2) \in Z - S,
\end{aligned}
$$

Problem 2:

$$
\begin{aligned}
&\vartheta_{,3}(x_1, x_2, 0^+) = -\frac{q_0(x_1, x_2)}{K}, \quad (x_1, x_2) \in S, \\
&\vartheta(x_1, x_2, 0^+) = 0, \quad (x_1, x_2) \in Z - S, \\
&\sigma_{33}(x_1, x_2, 0^+) = 0, \quad (x_1, x_2) \in Z, \\
&\sigma_{31}(x_1, x_2, 0^+) = \sigma_{32}(x_1, x_2, 0^+) = 0, \quad (x_1, x_2) \in S, \\
&w_1(x_1, x_2, 0^+) = w_2(x_1, x_2, 0^+) = 0, \quad (x_1, x_2) \in Z - S,
\end{aligned}
$$

where Z denotes the entire $x_1 x_2$-plane. In addition, the usual requirements at infinity, namely, vanishing of the temperature and displacements in Problem 1 and heat fluxes and stresses in Problem 2 are assumed.

It is shown how these problems are reduced to the classical mixed problems of potential theory. A general formulation in terms of singular two-dimensional integral and integro-differential equations is also presented.

Problem 1 is reduced to the mechanical counterpart in isothermal elasticity, involving the crack S under symmetrical normal loading (Mode I of crack deformation). It should be mentioned here that this problem does not require the evaluation of the temperature distribution ϑ.

Problem 2 (more involved) is reduced to its mechanical skew-symmetrical problem (Mode II and III of crack deformation).

It follows that the thermoelastic fields and stress intensity factors of the problems under consideration can be obtained from their mechanical counterparts provided that the corresponding solutions for the isotropic elastic materials are available.

Paper [5] demonstrates how to implement this method for the solution to the problem of an interface insulated plane crack obstructing a uniform heat flux in a two-layered microperiodic space.

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