MODELING OF ENVIRONMENT ASSISTED DELAMINATION
I. QUASISTATIC CASE

Alla V. Balueva
Spelman College, Department of Mathematics, 350 Spelman Lane SW, Atlanta, GA 30314, USA

Summary A theoretical model of quasistatic growth of delaminations of a protection covering of metal due to diffusion of dissolved in metal atomic hydrogen into the formed cavity is considered. For the case of a penny-shaped delamination the main equation describing its growth is derived.

INTRODUCTION

As is well known, in conditions of hydrogen embrittlement, hydrogen reduces the fracture resistance of many metals and steels. Hydrogen absorbed by a metal is usually dissolved in the lattice in the *proton* form (e.g., Turnbull, 1993; Vehoff, 1997). Some of the protons reach the surface of pre-existing cracks where they recombinate with electrons and form *molecular* hydrogen in the crack cavity, which leads to accumulation of gas hydrogen inside the crack. The appearing excessive pressure can be especially dangerous when the metal surface is coated by a protective material (e.g., Nayyar, 1992) that does not allow hydrogen diffusion. For example, in the case of pipelines for hydrocarbon transport, anti-corrosion, polymer coating sometimes results in more frequent appearance of small scale delaminations (Gapharov et al., 1998). In this paper, a model of a penny-shaped delamination controlled by a diffusion process is considered. Kinetic equations describing the delamination growth and resulting in the expressions for the crack size and velocity of growth are derived.

FORMULATION OF MODEL. DERIVING OF THE MAIN KINETICS EQUATION

Let a half-space (a substrate of the base metal) saturated by gas with a concentration c_∞ at infinity be covered by a thin infinite layer circular delamination of thickness, h, and initial radius, 1_0, that appeares in the plane of connection, $z=0$, at the moment of $t=0$ (Figure 1). The covering layer is assumed to be thin as compared to the half-space

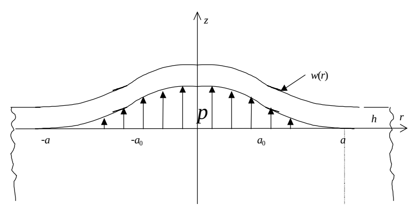

Figure 1. Delamination growth under the pressure of accumulated gas.

$D_0 = \frac{Eh^3}{12}$ is the plate flexural rigidity, h is the plate thickness.

Thus, if the typical size, l, of the delamination is smaller then its thickness, h, the major part of the strain energy is due to the deformation (bending) of the delaminated covering. Therefore, it is permissible to neglect the deformation of a substrate and to model it by the ideally rigid half-space from which a thin plate is delaminated (e.g. Balueva, 1999).

Once the gas is accumulated in the delamination, the critical condition is achieved on the crack contour and the delamination begins growing. The delamination crack is modeled by an ideal drain: we believe that the pressure inside the crack is sufficiently small so as not to achieve an equilibrum statement, and therefore, the molecular hydrogen accumulates inside the crack (see e.g. Goldstein, 1977). Then we can write the boundary value problem for finding of the concentration of atomic hydrogen, $c(r,z,t)$, at the moment of time, t, as the following:

$$\begin{aligned} &\Delta c = 0\ z \le 0 \\ &c\big|_{z=0} = 0, r \le l(t) \\ &\partial c/\partial z\big|_{z=0} = 0, r > l(t) \\ &c\big|_{z=\infty} = c_\infty \end{aligned} \qquad (1)$$

from where the density q of diffusion flux into a crack can be obtained in the following form

$$q(r,t) = -D\partial c/\partial z\big|_{z=0} = \frac{2}{\pi}\frac{c_\infty D}{\sqrt{l^2 - r^2}}, r \le l(t) \qquad (2)$$

Then a full gas flux into the crack can be written as follows:

$$\partial n/\partial t = -2\pi D\int_0^l \partial c/\partial z\Big|_{z=0}(r,t)r\,dr = 4c_\infty D l(t)\,, \qquad (3)$$

where n is the amount of gas moles in the crack.
The opening of the delamination under applied load p can be determined under the approximation of thin plates in the form

$$w(r) = \frac{pl^4}{64D_0}\left[1 - 2\left(\frac{r}{l}\right)^2 + \left(\frac{r}{l}\right)^4\right] \qquad (4)$$

and the potential energy of a circular plate can be written as follows:

$$U = \frac{D_0}{2}\int_0^l\int_0^{2\pi}(\nabla^2 w)^2 r\,dr\,d\varphi \qquad (5)$$

where

$$\nabla^2 w = \frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr} = \frac{pl^2}{8D_0}\left(2\frac{r^2}{l^2} - 1\right) \qquad (6)$$

Substituting (6) into (5), we obtain the following expression for energy:

$$U = -\frac{\pi}{6\cdot 64}\frac{p^2 l^6}{D_0} \qquad (7)$$

The critical energy released rate can be written in the following form:

$$\gamma_{scc} = -\frac{1}{2\pi l}\frac{\partial U}{\partial l} = \frac{1}{2\cdot 64}\frac{p^2 l^4}{D_0} \qquad (8)$$

The molecular hydrogen state in a crack is assumed to be described by the equation for ideal gas and all processes are considered to be isothermal. We have the expression for the volume of a crack in the form

$$V = \int_0^l\int_0^{2\pi} w(r) r\,dr\,d\varphi = \frac{\pi}{6\cdot 32}\cdot\frac{pl^6}{D_0} \qquad (9)$$

Taking into account (3), the equation of Mendeleev-Klaiperon can be written in the form

$$pV = 4c_\infty DRT\int_0^t l(t)dt \qquad (10)$$

where R is the universal gas constant and T is the gas temperature.
Substituting expression (9) for volume into (10) and taking into account (8), we obtain the main equation for the analysis of quasistationary growth of a penny-shaped delamination:

$$\frac{2\pi}{3} l^2 \gamma_{scc}^2 = 4c_\infty DRT\int_0^t l(t)dt \qquad (11)$$

from where after differentiating with respect to t both parts of equation (11) the delamination growth velocity can be written in the form

$$\dot{l} = \theta c_\infty / 2\gamma_{scc}\,, \qquad \theta = \frac{6}{\pi}RTD \qquad (12)$$

The expression (12) reveals that the delamination under these conditions spreads with a constant velocity.
Author would like to thank Dr. Leonid Germanovich for inestimable help and support, and would hope that joint continuation of the paper, II. Transient Case, is going to appear in press soon.

References

Toribio, J. and Kharin, V. (1997). The Effect of History on Hydrogen Assisted Cracking: 1. Coupling of Hydrogenation and Crack Growth. Int. J. Fract. **88**, 233-245.
Turnbull, A. (1992). Test Methods for Environment Assisted Cracking. British Corrosion Journal **27**, 271-289.
Vehoff, H. (1997). Hydrogen Related Material Problems, *Hydrogen in Metals III, Properties and Applications*, Springer-Verlag, Berlin Heidelberg New York, 348.