ON ASYMPTOTIC METHOD OF STATIC AND DYNAMIC BOUNDARY PROBLEMS SOLUTION OF ELASTICITY THEORY FOR THIN BODIES

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EXTENDED SUMMARY

Thinwalled bodies (bars, beams, plates, shells) own the very important property that one of their geometrical sizes essentially differs from the other two. By virtue of that, if in the corresponding equations of elasticity theory we pass to dimensionless coordinates, then the system of the transformed equations will have a small parameter and for the solution of the systems like that it is natural to use asymptotic methods. Meanwhile, the new system of the equations turned to be singularly perturbed and it is impossible to find its solution by a direct decomposition by small parameter. The solution of the corresponding boundary problem of elasticity theory consists of two qualitatively various types of solutions – solution of the inner problem and solution of the boundary layer. In the talk the ways of constructing these solutions and their conjunctions are described. Besides the classical boundary problem for thin bodies (on the facial surfaces the values of the stresses tensor components are given) nonclassical, from the position of plates and shells theories, boundary problems (on the facial surfaces the displacement vector or mixed conditions are given) are considered.

For the determination of solution Q of the inner problem the correct determination of asymptotic exponents of sought values is very important, since, as a rule not all the components of the stresses tensor and the displacement vector have the same contribution in the stress strain state. The solution of the inner problem is sought in the form of

$$Q(\xi,\zeta)=\varepsilon^{-q_a+s}Q^{(s)}(\xi,\zeta), \qquad s=\overline{0,N} \tag{1}$$

where Q is any of the stresses and displacements, q_a characterizes the asymptotic exponent of the given value. They are different for different values, their valuations react on the types of the boundary conditions on the facial surfaces. For example, in case of a plane problem for a rectangular area, if on the longitudinal borders of the rectangle, conditions of the first boundary problem of elasticity theory are given, we have

$$q=2 \text{ for } \sigma_{xx},u;\ q=1 \text{ for } \sigma_{xy};\ q=0 \text{ for } \sigma_{yy};\ q=3 \text{ for v} \tag{2}$$

and if on the longitudinal borders conditions of the second boundary problem (displacement vector) are given, then

$$q=1 \text{ for } \sigma_{xx},\sigma_{xy},\sigma_{yy};\ q=0 \text{ for } u,\text{v}. \tag{3}$$

Substituting (1) into the transformed equations of elasticity theory and taking into account (2) or (3), a noncontradictory system for a consequent determination of decomposition $Q^{(s)}$ coefficients is obtained. Analogous (1)-(3) decompositions for plates and shells are found. The connection of the solution of the inner problem with the solutions on Bernoulli-Coulomb classical theory of beams, Kirchhoff-Love theory of plates and shells, with precise theories on the base of softened hypotheses is displaced. The denoted approach is applied as well for thin layered structures.

The solution for a boundary layer is constructed; it is shown that in case of the first boundary problem for a rectangle of boundary layer $\sigma_{xxb},\sigma_{xyb}$ stress along the rectangle hight are self-balanced. The connection of the boundary layer with Saint-Venant principle is found, it is proved that in case of the first boundary problem for a rectangle, Saint-Venant principle is fulfilled mathematically precisely.

In general case the procedure of conjugation of the inner problem solutions and the boundary layer is described.

The asymptotic method permits us to find solutions of new classes of boundary problems of elasticity theory for anisotropic plates and shells. The consideration of these problems for layered

structures made it possible to establish connection between the solutions on elasticity theory and on applied models of foundations and bases, to reduce the calculation formula of the bed coefficient for a layered base.

The method turned out to be especially effective for the solution of nonclassical dynamic boundary problems. Free and forced vibrations of thin bodies on the base of dynamics equations of elasticity theory are considered. Connections between the values of free vibrations frequencies and the propagation velocities of elastic shear and longitudinal waves are established.

References

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