DAMAGE FIELD IDENTIFICATION USING FULL-FIELD DISPLACEMENT MEASUREMENTS

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Summary It is proposed to determine damage parameters in two dimensions (surface of a material) or three dimensions (in the bulk of a solid) by using full-field displacement measurements. A finite-element approach is developed to evaluate piece-wise constant elastic parameters modeled by an isotropic damage variable. Two sets of examples are discussed. The first series deals with mechanical fields obtained by finite element simulations to assess the performance of the approach. The second series is concerned with displacement measurements performed during a biaxial test on a composite material.

INTRODUCTION

The current development of reliable displacement field measurement techniques enables one to better characterize the behavior of materials and the response of structures to external loadings. Homogeneous materials under complex load histories or heterogeneous microstructures induce kinematic fields that require full-field analyses to understand the interactions between the material microstructure and the external loading. Full-field measurements often need inversion techniques to determine the mechanical properties field of the materials. Updating techniques based upon the constitutive equation error [1, 2, 3] have been used in the analysis of vibrations [4], the determination of damage fields [5] or to study heterogeneous tests (*e.g.,* Brazilian test [6]). Similarly, the so-called virtual field method has been used to identify homogeneous properties of composites [7, 8] (*i.e.,* in anisotropic elasticity). Another procedure is based upon the reciprocity gap [9] that can also be used to determine the local elastic field or to detect cracks in elastic media [10]. An alternative method that only needs displacement field data is proposed. In the present case, the in-plane displacements are measured on surfaces of samples by digital image correlation. It therefore consists in identifying elastic property fields on the surface of a sample.

THE EQUILIBRIUM GAP METHOD

An identification formulation is derived in which the displacements $\boldsymbol{u}(\boldsymbol{x})$ are *known* and the elastic properties are unknown. This problem setting is unconventional in the sense that classical FE formulations assume known mechanical properties and try to determine the displacement field for different types of boundary conditions. The potential energy theorem allows for a weak formulation of the equilibrium equations, which is linearly dependent on the displacements *and* elastic properties. Quadratic square elements are considered for which each node corresponds to a measurement point. This hypothesis allows us to derive a specific formulation in which only middle nodes are considered. When the damage parameter D_e is constant for a given element e occupying a domain Ω_e, the elementary stiffness matrix can be written as

$$[\boldsymbol{K}_{me}](D_e) = (1 - D_e)[\boldsymbol{K}_{me0}] \tag{1}$$

where $[\boldsymbol{K}_{me0}]$ is the elementary stiffness matrix of an undamaged element. Similarly, the strain energy E_{me} can be written as

$$E_{me}(D_e, \{\boldsymbol{u}_e\}) = \frac{1 - D_e}{2}\{\boldsymbol{u}_e\}^t[\boldsymbol{K}_{me0}]\{\boldsymbol{u}_e\}, \tag{2}$$

where $\{\boldsymbol{u}_e\}$ is the nodal displacement column vector and ^{t} the matrix transposition. In the absence of external load on the considered nodes, a residual force $\boldsymbol{F}_r$ for each middle node '$\pm$' of two neighboring elements '$-$' and '$+$' arises when equilibrium is not satisfied

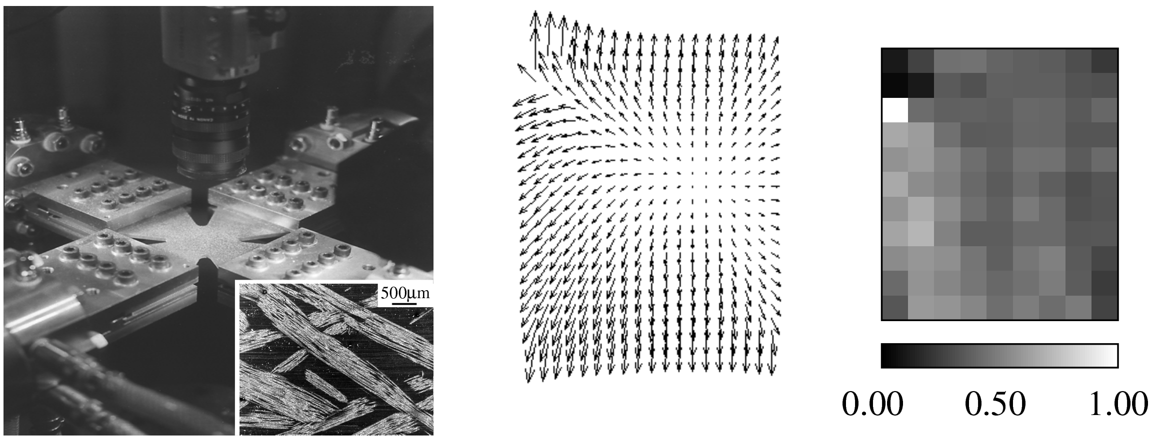
$$\boldsymbol{F}_r = \frac{\partial E_{m\pm}}{\partial \boldsymbol{u}_\pm}(D_e^-, D_e^+, \{\boldsymbol{u}_e^-\} = \{\boldsymbol{u}_m^-\}, \{\boldsymbol{u}_e^+\} = \{\boldsymbol{u}_m^+\}), \tag{3}$$

with $E_{m\pm}(D_e^-, D_e^+, \{\boldsymbol{u}_e^-\}, \{\boldsymbol{u}_e^+\}) = E_{me}(D_e^-, \{\boldsymbol{u}_e^-\}) + E_{me}(D_e^+, \{\boldsymbol{u}_e^+\})$, where $\boldsymbol{u}_\pm$ is the displacement vector of the considered middle node '$\pm$', D_e^-, D_e^+ are the damage variables in elements '$-$' and '$+$', respectively. In Eqn. (3), the nodal displacements $\{\boldsymbol{u}_e^-\}$ and $\{\boldsymbol{u}_e^+\}$ are equal to the corresponding measurements $\{\boldsymbol{u}_m^-\}$ and $\{\boldsymbol{u}_m^+\}$. The equilibrium gap method consists in minimizing a norm of the residuals $\boldsymbol{F}_r$ with respect to the unknowns D_e^- and D_e^+. By considering all the equilibrium equations, a global (over-determined) linear system is obtained [11]. Based upon the residuals, local error indicators are introduced. Of those, a whole family can be defined when the exact solution is unknown.

APPLICATIONS

A first series of computations considers a structure discretized with quadratic elements. Three edges are clamped and, in the middle of the fourth one, a point-force is applied. A uniform damage distribution is chosen and varies between 0 and 0.9. The first step consists in running a conventional FE analysis. The nodal displacements (*i.e.*, the “measurements") are stored and constitute the inputs to the damage identification procedure. The aim is to use the non-standard FE formulation to recover the damage field. Since the damage field is known, an RMS error between the prescribed and identified damage values can be used. For more than 2400 elements, a maximum error of 5.1% is observed for all the analyzed configurations [11].

Second, a vinylester matrix reinforced by E glass fibers is studied. A quasi-uniform distribution of orientations leads to an isotropic elastic behavior prior to matrix cracking and fiber breakage, which are the main damage mechanisms. A cross-shaped specimen is loaded in a multiaxial testing machine (Fig. 1-a). The experiment is performed in such a way that the forces applied along two perpendicular directions are identical. The displacement field of Fig. 1-b is measured by digital image correlation. Each “measurement point” corresponds to the center of an interrogation window of size 64×64 pixels, equivalent to a surface of about 8 mm^2. At this scale, the material is not homogeneous (see Fig. 1-a). The shift between two neighboring measurement points is 32 pixels. A sub-pixel algorithm is used. It enables for a displacement resolution of a few hundredths of one pixel for 8-bit pictures. To achieve a better robustness, an iterative and multi-scale version was used [12]. Figure 1-c shows the damage field computed from the measured displacement field. From the analysis of Fig. 1-b, a crack clearly appears on the top left corner for the last load level before failure. This crack can be observed by the three dark elements.



-a- -b- -c-

Figure 1. a-Sample in the testing machine ASTRÉE and microstructure of the studied composite. b-Displacement fields measured by digital image correlation for one load level close to failure. c-Corresponding damage field $(1 - D)$ identified with the proposed approach.

References

[1] Ladevèze P. Technical report Aérospatiale, les Mureaux (France), **33.11.01.4**, 1983.
[2] Kohn R.V., Lowe B.D. *Math. Mod. Num. Ana.* **22**: 119-158, 1988.
[3] Bui H.D., Constantinescu A. *Arch. Mech.* **52**: 511-522, 2000.
[4] Ladevèze P., Nedjar D., Reynier M. *AIAA* **32**: 1485-1491, 1994.
[5] Geymonat G., Hild F., Pagano S. *C. R. Mecanique* **330**: 403-408, 2002.
[6] Calloch S., Dureisseix D., Hild F. *Technologies et Formations* **100**: 36-41, 2002.
[7] Grédiac M. *C. R. Acad Sci. Paris* **309** [Série II]: 1-5, 1989.
[8] Grédiac M., Toussaint E., Pierron F. *C. R. Mecanique* **330**: 107-112, 2002.
[9] Bui H.D. *C. R. 2^e Colloque national de calcul des structures*. Hermès, Paris **1**: 25-35, 1995.
[10] Andrieux S., Ben Abda A., Bui H.D. *Inverse Problems* **15**: 59-65, 1999.
[11] Claire D., Hild F., Roux S. *C. R. Mecanique* **330**: 729-734, 2002.
[12] Hild F., Raka B., Baudequin M., Roux S., Cantelaube F. *Appl. Optics* **IP 41**: 6815-6828, 2002.