MICROMECHANICAL MODELLING OF THE DEFORMATION AND DAMAGE OF INELASTIC BRITTLE THREE-PHASE COMPOSITES: APPLICATION TO FIBER-REINFORCED CONCRETE

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Summary : This paper deals with some aspects of the modelling, simulation and experimentation of the deformation and damage of a class of non-linear three-phase brittle composite materials. To this end, Mori-Tanaka micromechanical model is used. The predictions obtained by our model are validated against finite element simulations and experimental results performed on Steel Fiber-Reinforced Concrete specimens.

Introduction

In the last decades, composite materials have occupied a great place in industrial applications. Consequently the study and the prediction of their mechanical behaviour have taken a crucial importance. In the present study, we have focused our work on some aspects of the modelling, simulations and experimentation of a class of inelastic brittle three-phase composite materials.

Micromechanical modelling

The aim of this section is to predict the mechanical response of an inelastic matrix containing perfectly bonded elastoplastic inclusions (spheres or ellipsoids) and cavities by means of the Mori-Tanaka homogenization scheme. To this end, we consider our three-phase composite as an aggregate made of two grains, one containing inclusions and matrix material, the other one containing cavities also embedded in the matrix phase. Next, each grain is homogenized using Mori-Tanaka model. Finally the set of the two homogenous grains is itself homogenized following Voigt scheme. After some algebraic developments we obtain the following expression of the homogenized non-linear tangent operator which relates the macro-strain to the macro-stress rates.

$\overline{C}(t) = C^{nl}_{ref}(t) + v_1 [C^{nl}_1(t) - C^{nl}_{ref}(t)] : A_1 - v_2 C^{nl}_{ref}(t) : A_2$ where $A_{1,2}$ are the strain concentration tensors for each phase and C^{nl}_{ref} refers to the stiffness of a reference material which represents the matrix phase. We also assume that all non-linear operators are spatially uniform and vary with time.

Constitutive models

The damage in the matrix phase which occurs in the form of microcraks growth and coalescence, is accommodated with the help of Ju's model [2]. It is an energy-based damage model which the damage in the material is linked to the history of elastic plastic state variables. In our study, we only consider the case of elasticity coupled with damage. In summary, the model proposed by Ju gives the following expression of the damaged stiffness:

$$C^d_{t_{n+1}} = \begin{cases} C^d_{t_n} & \text{if } \widetilde{G} - r \leq 0 \\ C^d_{t_{n+1}} - \left(\xi_{t_{n+1}} - \xi_{t_n}\right) H_{t_{n+1}} P^+_{t_{n+1}} : C^0 : P^+_{t_{n+1}} & \text{if not} \end{cases}$$

One can see from this equation that the stiffness is governed by a competition between available energy "*G*"and suitable energy for damage "*r*". "*H*" represents the damage evolution function, $\boldsymbol{\xi}$ is a characteristic damage measure, C^0 is the elastic stiffness and P^+ is a positive projection tensor. This model also assumes a spectral decomposition of the strain tensor to take into account the anisotropic damage. Since the composite is assumed to be damaged only under tensile strains, the expression of $\boldsymbol{\xi}$ takes the following form:

$\xi = \frac{1}{2} \varepsilon^+ : C^0 : \varepsilon^+$. The elastoplastic behaviour of the reinforcement is described by the classical J_2 plasticity theory. To handle those constitutive models in our micromechanical modelling, we have developed a robust numerical algorithm for an incremental tangent formulation which enables us to predict and simulate within reasonable CPU time and memory the response of such composite materials. In this algorithmic part we have pointed out the crucial issue of the computation of the Eshelby tensor [1]. We have resolved this problem by extracting the isotropic part of the anisotropic non-linear operator of the matrix phase. From this isotropic part we compute the instantaneous Poisson's coefficient which is needed for Eshelby's tensor computation. There are many ways to perform this extraction[1], in this work we use the following one:

$$C^{iso} = \frac{1}{3} C^{ani}_{lljj} I^{vol} + \frac{1}{5}\left(C^{ani}_{lljj} - \frac{1}{3} C^{ani}_{lljj}\right) I^{dev} \qquad \nu = \frac{C^{iso}_{1122}}{C^{iso}_{1111} + C^{iso}_{1122}}$$

We have seen that Ju's model is developed in a secant formulation and gives a secant damaged operator, to agree with our incremental formulation we have used the perturbation technique in order to compute the tangent damaged operator.

Experimental study

As an application, we have studied the mechanical behaviour of Steel Fiber Reinforced Concrete (SFRC). Several specimens of concrete and fiber-reinforced concrete with different volume fractions of fibers were manufactured. Those specimens were tested under four point bending. The tests on concrete specimens have allowed us to fit the parameters of Ju's model, the others were used in the validation step of our numerical code. The first results obtained seem to indicate that when fibers are added to concrete with a given volume fraction, the porosity increases and becomes significant. This lead us to consider our initially two-phase composite as a three phase one with cavities as the third phase. To make those experimental results useful, we also have developed a tangent formulation and it's corresponding numerical algorithm to allow us the construction of the stress-strain curve from the load-deflection one in the non-linear regime. The key idea of this formulation is to relate the increment of bending moment to the increment of the normal stress for a given cross section.

$$M_i = B\left[\int_0^{H-C} \sigma_i \, y dy + \int_{-C}^{0} \sigma_i \, y \, dy\right]$$ where B and H are the geometric characteristics of the section and C the position of the neutral axis and the stress at the i th increment is given by: $\sigma_i = \sigma_{i-1} + k_i \, (\varepsilon - \varepsilon_{i-1})$

Numerical simulations

The efficiency of our code is evaluated through numerical simulations compared against both 2D and 3D F.E simulations fig.1 and experimental results. Several discriminating tests were simulated, fig. 2 show an example.of FRC with 10% of volume fraction of fibers under cyclic tensile and compressive loading. We also validated our numerical predictions against our experimental results.

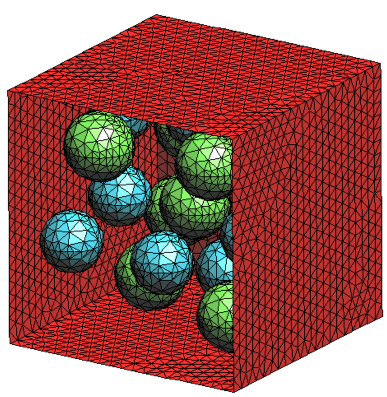

Fig. 1. 3D F.E simulations on a periodic cubic cell

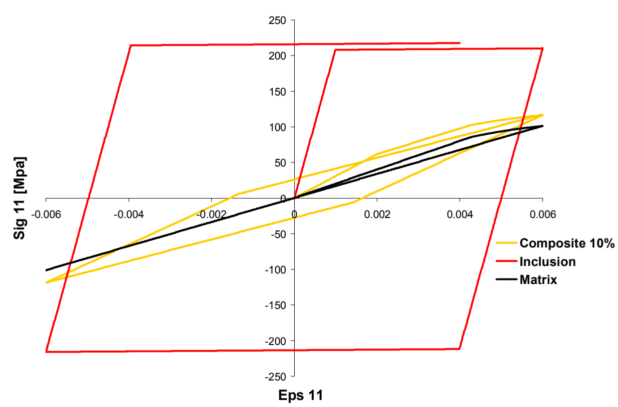

Fig. 2. Steel Fiber-Reinforced Concrete under cyclic loading

References

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