DAMAGE ACQUIRED ANISOTROPY IN ELASTIC-PLASTIC MATERIALS

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Summary Damage coupled model of elastic-moderate plastic material based on the concept by Hayakawa-Murakami is developed. Incremental matrix constitutive equations for plane stress conditions, that account for damage anisotropy and crack opening/closing effect, are explicitly derived and implemented to user subroutine of ABAQUS finite element code. Numerical examples illustrate consecutive stages of elastic-damage and plastic-damage as well as stiffness recovery on reverse loading.

A thermodynamically consistent framework for elasto-plasticity coupled with damage, based on existing state and dissipation coupling models, is discussed. Weak dissipation coupling, following a concept of existence of two dissipation potentials, plastic $F^{\mathrm{p}}(\boldsymbol{\sigma},R)$ and damage $F^{\mathrm{d}}(\mathbf{Y},B)$, expressed in the space of thermodynamic forces associated with the plasticity $(\boldsymbol{\varepsilon}^{\mathrm{p}},r)$ and damage $(\mathbf{D},\beta)$ variables, is focused. Crack opening/closing response to reverse loading cycles by the use of generalized projection operators that extends the Hansen and Schreyer [1] idea via the additional material constant $\zeta \in \langle 0,1 \rangle$, $\overline{\mathbf{P}}_{\varepsilon/\sigma} = \mathbf{P}^{+}_{\varepsilon/\sigma} + \zeta \mathbf{P}^{-}_{\varepsilon/\sigma}$, is incorporated. This approach allows for effect of negative principal components of strain or stress tensor on damage evolution, as observed in brittle materials (cf. Murakami and Kamiya [2]).
The elastic-plastic-damage constitutive equations postulated in a total form and calibrated for spheroized graphite cast iron by Hayakawa and Murakami [3] are adopted. They are based on the assumption of the Gibbs state potential

$$\Gamma(\boldsymbol{\sigma},r,\mathbf{D},\beta) = \boldsymbol{\sigma} : \boldsymbol{\varepsilon} - \psi(\boldsymbol{\varepsilon}^{\mathrm{e}},r,\mathbf{D},\beta) \tag{1}$$

where ψ denotes the Helmholtz free energy per unit mass. based on the classical scheme the elastic strain and the conjugate forces are

$$\boldsymbol{\varepsilon}^{\mathrm{e}} = \frac{\partial \Gamma}{\partial \boldsymbol{\sigma}} = {}^{\mathrm{s}}\mathbf{C}^{\mathrm{e}}(\mathbf{D}) : \boldsymbol{\sigma}\,,\ R = \frac{\partial \Gamma}{\partial r}\,,\ \mathbf{Y} = \frac{\partial \Gamma}{\partial \mathbf{D}}\,,\ B = \frac{\partial \Gamma}{\partial \beta} \tag{2}$$

where ${}^{\mathrm{s}}\mathbf{C}^{\mathrm{e}}(\mathbf{D})$ stands for the effective, secant elastic-damage compliance tensor.
The dissipation potential is composed of two coupled parts

$$F(X_m; r,\mathbf{D},\beta) = F^{\mathrm{p}}(\boldsymbol{\sigma},R;\mathbf{D}) + F^{\mathrm{d}}(\mathbf{Y},B;\mathbf{D},r,\beta) \tag{3}$$

hence, when the extended normality rule is assumed both for plastic and damage surfaces, the plasticity and damage fluxes are controlled by two Lagrange multipliers $\dot{\lambda}^{\mathrm{p}}$ and $\dot{\lambda}^{\mathrm{d}}$

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = \dot{\lambda}^{\mathrm{p}} \frac{\partial F^{\mathrm{p}}}{\partial \boldsymbol{\sigma}}\,,\ \dot{r} = \dot{\lambda}^{\mathrm{p}} \frac{\partial F^{\mathrm{p}}}{\partial(-R)}\,,\ \dot{\mathbf{D}} = \dot{\lambda}^{\mathrm{d}} \frac{\partial F^{\mathrm{d}}}{\partial \mathbf{Y}}\,,\ \dot{\beta} = \dot{\lambda}^{\mathrm{d}} \frac{\partial F^{\mathrm{d}}}{\partial(-B)} \tag{4}$$

The Khun-Tucker relations are used in order to specify loading/unloading conditions for plasticity and damage, respectively

$$\dot{\lambda}^{\mathrm{p/d}} \geq 0\,,\ F^{\mathrm{p/d}} = 0\,,\ \dot{\lambda}^{\mathrm{p/d}} F^{\mathrm{p/d}} = 0\,. \tag{5}$$

In order to derive elastic-damage equation in the incremental form

$$\{\dot{\boldsymbol{\varepsilon}}^{\mathrm{e}}\} = \mathbf{C}^{\mathrm{e}}(\boldsymbol{\sigma},\mathbf{D},\mathbf{Y})\{\dot{\boldsymbol{\sigma}}\} \tag{6}$$

we follow procedure described in Kuna-Ciskał and Skrzypek [4] where the effective, tangent elastic-damage stiffness matrix was determined based on the relevant model of elastic-damage materials [2]. The effective, tangent elastic-damage compliance tensor is determined according to the scheme

$$\mathbf{C}^{\mathrm{e}}(\boldsymbol{\sigma},\mathbf{D},\mathbf{Y}) = {}^{\mathrm{s}}\mathbf{C}^{\mathrm{e}}(\mathbf{D}) + \frac{\partial\, {}^{\mathrm{s}}\mathbf{C}^{\mathrm{e}}(\mathbf{D})}{\partial \mathbf{D}} : \frac{\partial \mathbf{D}}{\partial \boldsymbol{\sigma}} : \boldsymbol{\sigma} \tag{7}$$

Plastic-damage strain increment is furnished as (cf. Bielski, Kuna-Ciskał and Skrzypek [5])

$$\{\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}}\} = \mathbf{C}^{\mathrm{p}}(\boldsymbol{\sigma},\mathbf{D},\mathbf{Y},r)\{\dot{\boldsymbol{\sigma}}\} \tag{8}$$

Finally, in frame of small strain theory, we end-up with the general elastic-plastic-damage constitutive equation in the incremental form

$$\{\dot{\boldsymbol{\varepsilon}}\} = \mathbf{C}(\boldsymbol{\sigma},\mathbf{D},\mathbf{Y},r)\{\dot{\boldsymbol{\sigma}}\} \tag{9}$$

with the local compliance matrix $\mathbf{C} = \mathbf{C}^{\mathrm{e}}(\boldsymbol{\sigma},\mathbf{D},\mathbf{Y}) + \mathbf{C}^{\mathrm{p}}(\boldsymbol{\sigma},\mathbf{D},\mathbf{Y},r)$ dependent on variables at the consecutive equilibrium point. When deriving equations (6), where unilateral damage effect is included by a concept of the modified stress tensor $\bar{\sigma}_{\mathrm{ij}} = B_{\mathrm{ijkl}}\sigma_{\mathrm{kl}}$ (cf. [3]), the main difficulty arises from the incremental transformation tensor $D_{\mathrm{ijkl}} = \dfrac{\partial \bar{\sigma}_{\mathrm{ij}}}{\partial \sigma_{\mathrm{kl}}}$, the explicit application of which is somewhat cumbersome.

Hence, following [4], in the present paper we apply simplified formulae for the matrices $\mathbf{B}$ and $\mathbf{D}$, in order to derive explicit form of the compliance matrix $\mathbf{C}$ in plane stress $\sigma_{33} = 0$ conditions $\boldsymbol{\sigma} = \{\sigma_{11}, \sigma_{22}, \sigma_{12}\}$. In case of plane stress the plane rotation by the angle α of the stress tensor from a current system σ_{kl} to principal directions σ_{I}, modification to $\bar{\sigma}_{\mathrm{I}}$ and backward transformation to the current system must be performed, such that, according to the procedure for complex function, the following holds

$$\frac{\partial \bar{\sigma}_{\mathrm{ij}}}{\partial \sigma_{\mathrm{kl}}} = B_{\mathrm{ijkl}} + \frac{\partial B_{\mathrm{ijpq}}}{\partial \alpha}\frac{\partial \alpha}{\partial \sigma_{\mathrm{kl}}}\sigma_{\mathrm{pq}} \tag{10}$$

Taking inverse of the matrix $\mathbf{C}$ the increments of stresses are calculated as

$$\{\dot{\boldsymbol{\sigma}}\} = \mathbf{C}^{-1}\{\dot{\boldsymbol{\varepsilon}}\} \tag{11}$$

where damage-coupled elastic-plastic stiffness $\mathbf{C}^{-1}$ is accepted as an approximation of the local Jacobian matrix of the constitutive model implemented to the ABAQUS finite element code. Effective algorithm for plastic and damage loading/unloading conditions based on the doubly-passive predictor-plastic/damage corrector approach is used. A more general, fully coupled return mapping computational algorithm, is due to Zhu and Cescotto [6].

Numerical examples illustrate capability of the model developed to simulate anisotropic damage effect on elastic and elastic-plastic response of simple structures under plane stress conditions. Partial stiffness recovery on reverse uniaxial loading cycles is also captured.

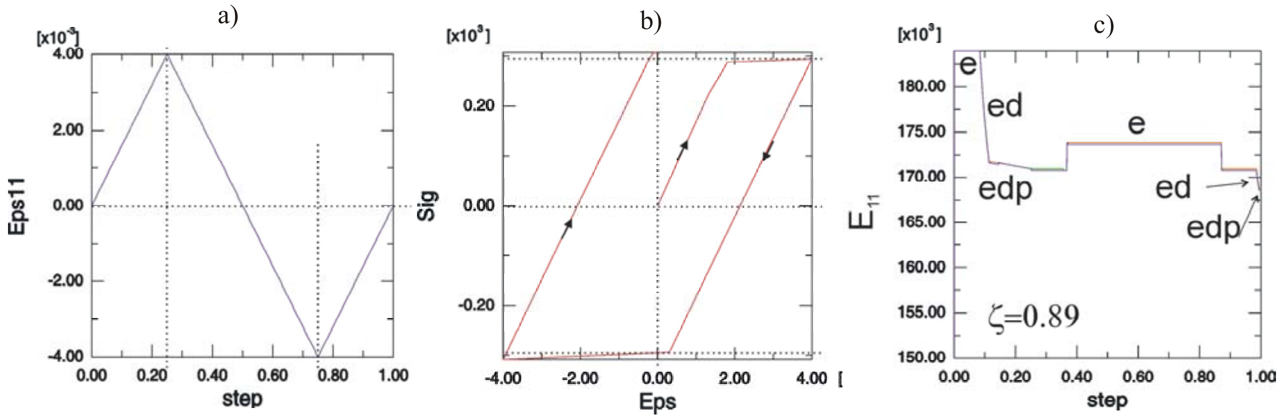

Fig.1.a) kinematic loading cycle; b) stress-strain hysteresis loop; c) elastic stiffness recovery

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