NUMERICAL ANALYSIS OF NONLOCAL ANISOTROPIC CONTINUUM DAMAGE

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Summary The present paper deals with the numerical analysis of large elastic-plastic deformation and localization behavior of anisotropically damaged ductile solids within the framework of nonlocal continuum mechanics. Estimates of the stress and strain histories are obtained from a numerical integration algorithm which employs an inelastic predictor followed by an elastic corrector step. Numerical simulations show the influence of several model parameters on the deformation and localization prediction.

INTRODUCTION

Large inelastic deformations of metals are usually accompanied by damage processes due to microdefects. Their proper understanding and their mechanical description are of importance in discussing the mechanical effects of the material deterioration on the macroscopic behavior of solids as well as in elucidating the mechanisms leading to final fracture. In this context, nonlocal effects seem to be important when deformation mechanisms governed by microscopic phenomena as well as scale effects are considered in order to explain and predict certain experimentally observed critical phenomena.

FUNDAMENTAL GOVERNING EQUATIONS

A nonlocal generalization of the framework presented by Brünig [1] is proposed to describe the inelastic deformation behavior of ductile metals. It is based on the introduction of respective Helmholtz free energy functions of fictitious undamaged and of current damaged configurations. In addition, to be able to minimize the analytical and numerical difficulties associated with general nonlocal formulations the nonlocal concept is applied only to those parameters which cause material softening while the elastic behavior is still assumed to be governed by a local formulation. Thus, the kinematic relations as well as the balance equations remain local, and the distribution of the stresses and displacements is still governed by the standard differential equations of equilibrium and the associated boundary conditions. Nonlocal effects, on the other hand, are described via additional length quantities which play the role of material parameters and are used to obtain weighted averages of the corresponding local plastic and damage variables over a material volume of the body. The effective specific free energy $\bar{\phi}$ of the undamaged matrix material is decomposed into an effective elastic and an effective plastic part

$$\bar{\phi} = \bar{\phi}^{el}\left(\bar{\mathbf{A}}^{el}\right) + \bar{\phi}^{pl}\left(\gamma, \hat{\gamma}\right), \tag{1}$$

where $\bar{\mathbf{A}}^{el}$ is the effective elastic strain tensor, γ and $\hat{\gamma}$ denote an internal plastic variable and its nonlocal counterpart, respectively. In addition, plastic yielding of the matrix material is described by the nonlocal yield condition

$$f^{pl}\left(\bar{I}_1, \bar{J}_2, c\right) = \left(1 - \frac{a}{c}\bar{I}\right)^{-1}\sqrt{\bar{J}_2} - c\left(\gamma, \hat{\gamma}\right) = 0 \ , \tag{2}$$

where $\bar{I}_1 = \mathrm{tr}\bar{\mathbf{T}}$ and $\bar{J}_2 = \frac{1}{2}\mathrm{dev}\bar{\mathbf{T}} \cdot \mathrm{dev}\bar{\mathbf{T}}$ are invariants of the effective stress tensor $\bar{\mathbf{T}}$, c denotes the strength coefficient of the matrix material and a represents the hydrostatic stress coefficient [2].
Moreover, the Helmholtz free energy function of the damaged material sample is assumed to consist of three parts:

$$\phi = \phi^{el}\left(\mathbf{A}^{el}, \mathbf{A}^{da}\right) + \phi^{pl}\left(\gamma, \hat{\gamma}\right) + \phi^{da}\left(\mu, \hat{\mu}\right) \quad . \tag{3}$$

Namely, the elastic free energy ϕ^{el}, which is an isotropic function of the elastic and damage strain tensors, $\mathbf{A}^{el}$ and $\mathbf{A}^{da}$, is used to describe the elastic response of the damaged material at the current state of deformation and material damage. Furthermore, the energies ϕ^{pl}, due to plastic hardening, and ϕ^{da}, due to damage strengthening, only take into account the respective internal state variables, γ and μ, as well as their nonlocal counterparts $\hat{\gamma}$ and $\hat{\mu}$ which are taken to be volume averages of γ and μ, respectively. In addition, evolution of damage is described by the nonlocal damage criterion

$$f^{da}\left(I_1, J_2, \tilde{\sigma}\right) = I_1 + \tilde{\beta}\sqrt{J_2} - \tilde{\sigma}\left(\mu, \hat{\mu}\right) = 0 \ , \tag{4}$$

where $\tilde{\beta}$ represents the influence of the deviatoric stress state on the damage condition and $\tilde{\sigma}$ denotes the equivalent damage stress measure.
On the numerical side a key factor in the numerical treatment of inelastic continuum models using the finite element method is the numerical integration of the nonlinear constitutive equations governing the flow and damage behavior as well as the evolution of internal state variables. Estimates of the irreversible strain histories are obtained via an extended

version of the inelastic predictor method. In the elastic corrector step the plastic and damage correctors, $\Delta_{er}\hat{\gamma}$ and $\Delta_{er}\hat{\mu}$, are expanded in respective Taylor series, e.g.

$$\Delta_{er}\hat{\gamma} = \Delta_{er}\gamma + d_{pl}\frac{\partial^2 \Delta_{er}\gamma}{\partial \mathbf{x}\cdot\partial \mathbf{x}} + \ldots = \Delta_{er}\gamma + d_{pl}\nabla^2\left(\Delta_{er}\gamma\right) + \ldots, \tag{5}$$

where only terms up to second order are retained and d_{pl} takes into account a plastic internal length scale [2]. This leads to the system of coupled elliptic partial differential equations for the estimates of the correctors of the equivalent plastic and damage strains

$$m_{pl}d_{pl}\frac{\partial c}{\partial \gamma}\nabla^2\left(\Delta_{er}\gamma\right) + \left[\sqrt{2}G_1k_1 + \frac{\partial c}{\partial \gamma}\right]\Delta_{er}\gamma + \sqrt{2}G_1k_2\Delta_{er}\mu = c_{pr} - c(t) \tag{6}$$

and

$$m_{da}d_{da}\frac{\partial \sigma}{\partial \mu}\nabla^2\left(\Delta_{er}\mu\right) + \left[\sqrt{2}G_2k_4 + \frac{\partial \sigma}{\partial \mu}\right]\Delta_{er}\mu + \sqrt{2}G_2k_3\Delta_{er}\gamma = \sigma_{pr} - \sigma(t). \tag{7}$$

where G_1 and G_2 as well as $k_1,\ldots,k_4$ represent modified material parameters and m_{pl} and m_{da} denote the relative weights of the nonlocal effects compared to the local ones. Equations (6) and (7) are solved via a standard finite difference method employing an overlay mesh defined by the Gaussian integration points of the underlying finite element mesh, whereas the global equilibrium equations are solved using standard displacement-based finite elements. At the end of each time step equivalent plastic and damage strain increments are computed simultaneously which then lead to the corresponding tensorial quantities employing an integration scheme with an exponential shift.

NUMERICAL EXAMPLES

The numerical simulations deal with the finite deformation and localization behavior of uniaxially loaded rectangular specimens with clamped ends. The corresponding load-deflection curves are shown in Fig. 1 based on a nonlocal elastic-plastic material model without damage, including isotropic damage and including anisotropic damage discussed above, respectively. In particular, Fig. 1 shows that with the onset of damage the numerical calculations including isotropic and anisotropic damage show rapid loss in load carrying capacity with increasing elongation of the specimen which agrees with experimental observations. Fig. 2 shows the evolution of the void volume fraction f with increasing equivalent plastic strain γ. The numerical calculation for the isotropic damage model shows an increase in void volume fraction after the onset of isotropic damage. The point where the two curves based on the isotropic and the anisotropic model separate characterizes the onset of anisotropic damage which leads to an even larger increase in void volume fraction with growing plastic strain.

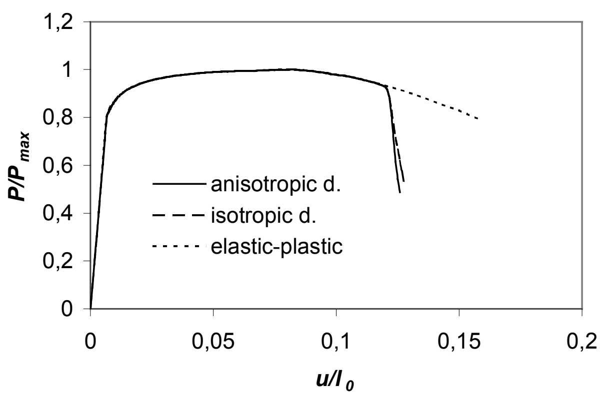

Fig. 1: Load-deflection curves

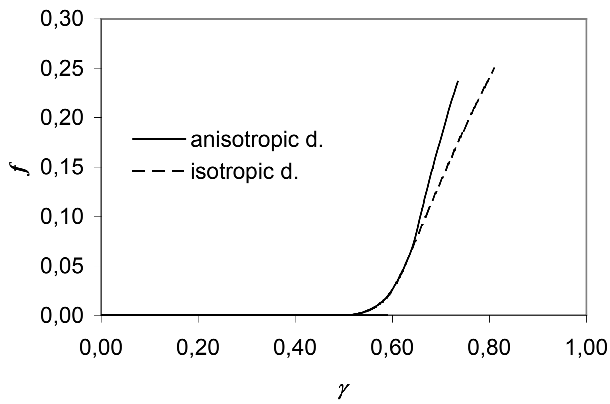

Fig. 2: Evolution of void volume fraction

CONCLUSIONS

The proposed large strain damage theory is a robust and efficient framework to develop structural models capable for providing practical solutions of general problems in engineering.

References

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[2] Brünig, M., Ricci, S., Obrecht, H.: Nonlocal large deformation and localization behavior of metals, Comp. Struct. **79**, 2063-2074, 2001.