STABILITY AND CREEP DAMAGE OF QUASI-BRITTLE MATERIALS

Noël Challamel, Christophe Lanos and Charles Casandjian
LMSM, INSA de Rennes, 20 avenue des Buttes de Coësmes, 35043 Rennes, France

Summary A time-dependent damage model is developed for quasi-brittle materials like rock or concrete, in the framework of Continuum Damage Mechanics. Phenomena like relaxation, creep and rate-dependent loading are covered using a unified framework. Creep failure under high-sustained load is explained quite simply within stability theory. Kachanov's equation is commented within the new approach. Creep failure appears as the manifestation of a bifurcation phenomenon.

INTRODUCTION

Geomaterials, such as rock or concrete are often called quasi-brittle materials. Unlike perfectly brittle materials, there are significant zones of softening damage in front of the generated crack tip. Softening is a subject that has profited from a very rich literature for now two decades. It is now well admitted that local damage softening models (which do not exhibit internal length) suffer of a pathological mesh-dependence in finite-element computations. Techniques known as regularisation were developed to restore the well-posedness of the problem. An internal length is generally introduced and constrains the new regularised system towards a family of imposed solutions (non local damage models).
Moreover, the long term performance of concrete or rock structures is fundamentally affected by the behaviour of the material after cracking. In order to simulate realistic concrete or rock behaviour, time-dependence of fracture has to be also considered. Understanding the interaction between the two phenomena (time-dependence and softening) is important to design reliable civil engineering structures subjected to high level and long time loading. As most studies applied to time-dependent concrete modelling have considered viscoelasticity, very few time-dependent damage models are available in the literature. A time-dependent damage model is presented in this study applied to quasi-brittle materials like rock or concrete. This three-dimensional model is introduced in the framework of Continuum Damage Mechanics and is based on strong thermodynamical arguments. The model is a generalisation of the uniaxial model we have previously proposed to model creep failure in compression [1]. Broad range of loading paths can be covered with such model. The constitutive model incorporates different properties in tension and compression. One of the most features of the model is the aptitude to predict creep failure under high-sustained load.

DAMAGE EVOLUTION

For the considered visco-elastic damage process, internal variables are chosen as the strain tensor $\underline{\underline{\varepsilon}}$ and a scalar damage variable D. The fundamental equations of this time-dependent isotropic damage model are :

$$\dot{D} = \frac{1}{\eta}\left\langle \frac{\tilde{\varepsilon} - \left(\chi(D) + \tilde{\varepsilon}_D\right)}{\tilde{\varepsilon}_c} \right\rangle^m ; \quad \underline{\underline{\sigma}} = (1-D)\lambda_0 tr\underline{\underline{\varepsilon}}\,\underline{\underline{1}} + (1-D)2\mu_0\underline{\underline{\varepsilon}} \tag{1}$$

where $\tilde{\varepsilon}$ is the equivalent strain of the model proposed by Mazars. m is a dimensionless parameter and η is a time constant. $\tilde{\varepsilon}_c$ is a characteristic strain. χ is a definite positive function. $\underline{\underline{\sigma}}$ is the stress tensor. λ_0 and μ_0 are the Lamé's coefficients of the undamaged material. Such a mathematical system can be justified within thermodynamical arguments. $\tilde{\varepsilon}_D$ defines the size of the initial elastic strain domain. As for endochronic models, it is assumed that the model has no yield surface for the monotonic test ($\tilde{\varepsilon}_D = 0$). In cases of uniaxial compression or uniaxial tension tests, it can be shown with these assumptions that Eq. (1) can be converted into a dimensionless differential system (the dimensionless variables are noted by stars) :

$$\frac{dD}{dt^*} = \left\langle \varepsilon^* - \chi^*(D) \right\rangle^m ; \quad \sigma^* = (1-D)\varepsilon^* \tag{2}$$

EQUILIBRIUM CURVE

The differential equation (2) can also be written as :

$$\frac{d\sigma^*}{dt^*} = -\varepsilon^*\left\langle \varepsilon^* - \chi^*\left(1 - \frac{\sigma^*}{\varepsilon^*}\right) \right\rangle^m + \frac{\sigma^*}{\varepsilon^*}\frac{d\varepsilon^*}{dt^*} \quad \text{for } \varepsilon^* \neq 0 \tag{3}$$

This is a true non-linear differential equation which can not be written using Boltzmann's integral of linear viscoelasticity. Asymptotic solutions of constant strain rate tests are well documented in the literature for these non-linear systems. The stress-strain response for extremely slow loading is simply obtained when the term that does not involve strain or stress rate (term in bracket) vanishes in equation (3). This response is sometimes called "equilibrium curve". It turns out that for very slow motions, the stress-strain relation converges towards a time-independent damage model, chosen as :

$$\varepsilon^* = \chi^*(D) = 2D \tag{4}$$

An apparent elastic domain is identified for constant strain rate tests, which reveals the competition between the internal kinetics of damage and the strain rate imposed on the material.

CREEP DAMAGE MODELLING

Creep tests are now studied within this model. After a high rate loading test, the stress is assigned to a constant value during the creep test :

$$\sigma^*(t^*) = \overline{\sigma}^* \quad \forall\, t^* > 0 \tag{5}$$

The differential equations which govern the evolution of the state variable is now written as :

$$\varepsilon^* = \frac{\overline{\sigma}^*}{1-D}\,; \quad \frac{dD}{dt^*} = \left\langle \frac{\overline{\sigma}^*}{1-D} - \chi^*(D) \right\rangle^m \tag{6}$$

Formally, this equation is not so far to the specific creep Kachanov model, which expresses the rate of damage only according to the effective stress (Kachanov, 1958). Damage equilibrium solutions of Eq. (6) is investigated using Liapounov's stability theory. It is shown in [2] that the set of equilibrium solutions of this inelastic system is continuous. The smallest equilibrium solution (when it does exist) is stable for sufficiently small creep stress value. A bifurcation phenomenon appears for a critical stress ($\overline{\sigma}^* = 0.5$). Creep failure under high-sustained load (tertiary creep of figure 2) is explained as a bifurcation by loss of equilibrium (figure 1). m is equal to *1* for the simulations.

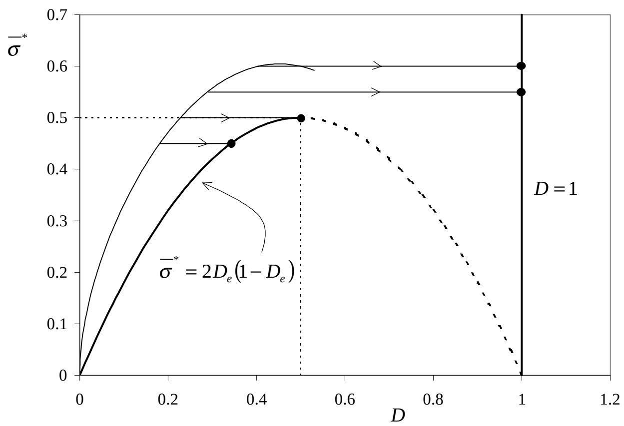

Fig. 1 – bifurcation diagram

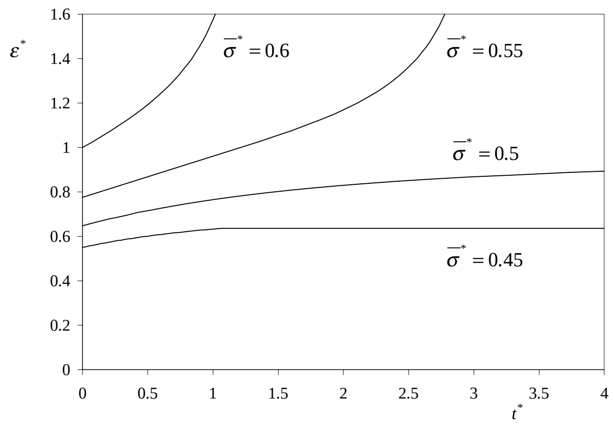

Fig. 2 – strain evolution for different creep stress

References

[1] Challamel N., Lanos C. and Casandjian C., Creep failure in concrete as a bifurcation phenomenon, *Int. J. Damage Mech.*, In Press, 2004.

[2] Challamel N. and Pijaudier-Cabot G., Stabilité et dynamique d'un oscillateur endommageable, *Revue Française de Génie Civil*, In Press, 2004.