FRICTIONAL AUTO-OSCILLATIONS UNDER THE ACTION OF ALMOST PERIODIC AND PERIODIC EXCITATIONS

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Summary Frictional auto-oscillations under the action both almost periodic and periodic forces are studied in this paper. The modulation equations are derived to study analytically the bifurcation behavior. The homoclinic Melnikov function is constructed to obtain the region of chaotic solutions of the modulation equations. In the case of the periodical force action and small dissipation a new approach for a construction of homoclinic trajectory of mechanical system is utilized. Also an approach to determine the onset of chaos based on some consequence from the Lyapunov stability definition is suggested for the case when dissipation is not small. Mutual instability of phase trajectories is used as a criterion of chaotic behavior in dynamical systems.

SYSTEM UNDER THE ACTION OF ALMOST PERIODIC FORCE

The mass-spring system interacting with moving belt is considered in this paper. The oscillations excited by both almost periodic and periodic forces are studied here. The multiple scales method is used to study the system under the action of the almost periodic force. The periodic motions and their bifurcations are investigated in the system of modulation equations. The homoclinic Melnikov function is constructed to analyze the chaotic states of the modulation equations. The analytical results are compared with the data of the numerical simulations.

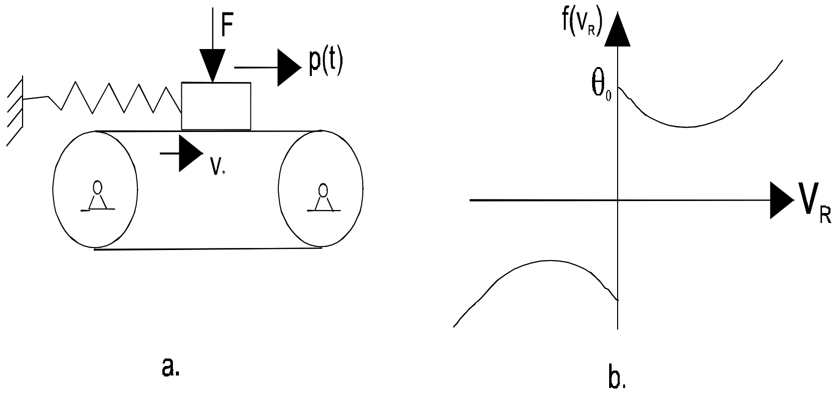

Fig.1. The mechanical system.

Fig.1 shows the system under consideration. The oscillations are described by the generalized coordinate ξ. The differential equation of oscillations is the following:

$$\xi'' + \xi = \varepsilon\left\{-\lambda\xi^3 + \gamma_1\cos\Omega_1\tau + \mu\left[\gamma_2\cos\Omega_2\tau - \tilde{\theta}P(\xi' - v_*)\right]\right\} \qquad (1)$$

where $P(\xi' - v_*) = T_0 sign(\xi' - v_*) - \alpha(\xi' - v_*) + \beta(\xi' - v_*)$ is nonlinear friction characteristics; ε and μ are two small parameters $0 \prec \varepsilon \prec\prec \mu \prec\prec 1$. Let us consider the following resonance: $\Omega_1 = 1 + \varepsilon\sigma; \Omega_2 \approx \Omega_1 + \varepsilon\Delta$. Using the multiple scales method, the following system of nonautonomous modulation equations is obtained

$$\rho' = \sqrt{\rho}\frac{\gamma_1}{\sqrt{2}}\sin\theta + \mu\left\{-\tilde{\theta}\alpha_1\sqrt{2\rho} + \rho\tilde{\theta}(\alpha - 3\beta v_*^2) - \frac{3}{2}\tilde{\theta}\beta\rho^2 + \sqrt{\rho}\frac{\gamma_2}{\sqrt{2}}\sin\theta\cos\Delta T_1 + \sqrt{\rho}\frac{\gamma_2}{\sqrt{2}}\cos\theta\sin\Delta T_1\right\};$$

$$\theta' = \sigma - \frac{3\lambda}{4}\rho + \frac{\gamma_1}{2\sqrt{2\rho}}\cos\theta + \mu\frac{\gamma_2}{2\sqrt{2\rho}}(\cos\theta\cos\Delta T_1 - \sin\theta\sin\Delta T_1). \qquad (2)$$

If μ=0 the system is Hamiltonian with heteroclinic orbits. The unperturbated system contains fixed motions. The almost periodic motions close to these fixed points and their bifurcations are studied analytically by the averaging method. The homoclinic Melnikov function is constructed to study the chaotic behavior of the modulation equations.

SYSTEM UNDER THE ACTION OF PERIODIC FORCE

A formation of a homoclinic trajectory (HT) is considered as a criterion of the chaos onset in dynamical systems under the action of periodic force. Mechanical system with a small periodic external excitation, nonlinear friction force and the Duffing type stiffness is governed by the following second order differential equation:

$$y'' - y + y^3 = f\cos(\omega t + \varphi) - P(y' - v_*). \tag{3}$$

Here a new approach for the HT construction in the nonlinear dynamical systems with phase space of dimensions equal to two in the case of small dissipation is proposed. Quasi- Padé approximants (QPA) are used for a representation of the time solution.

Multiplying the equation (3) by $y'(t)$ and integrating within the limits from $t = 0$ to $t = +\infty$ and from $t = 0$ to $t = -\infty$ we obtain the following equations where several integrals containing the small multipliers f, α, β and T_0 are integrated along the zero approximation trajectory ($\theta = 0, f = 0$) $y_0 = \sqrt{2}/ch(t)$:

$$\frac{a_0^2}{2} - \frac{a_0^4}{4} - \left(\alpha v_* - \beta v_*^3 - T_0\right)a_0 - \frac{2\alpha}{3} + \frac{8\beta}{35} + \frac{4\sqrt{2}\beta v_*}{5} + 2\beta v_*^2 + \\ + f\sin\varphi \int_0^{+\infty} \sin\omega t\, y_0'\, dt - f\cos\varphi \int_0^{+\infty} \cos\omega t\, y_0'\, dt = 0; \tag{4}$$

$$\frac{a_0^2}{2} - \frac{a_0^4}{4} - \left(\alpha v_* - \beta v_*^3\right)a_0 + \frac{2\alpha}{3} - \frac{8\beta}{35} + \frac{4\sqrt{2}\beta v_*}{5} - 2\beta v_*^2 - f\sin\varphi \int_0^{+\infty} \sin\omega t\, y_0'\, dt - \\ - f\cos\varphi \int_0^{+\infty} \cos\omega t\, y_0'\, dt - T_0 \int_{-\infty}^{0} sign\left(y_0' - v_*\right) y_0'\, dt = 0. \tag{5}$$

Here
$$\int_0^{+\infty} \sin\omega t\, y_0' dt = \int_{-\infty}^{0} \sin\omega t\, y_0' dt = -\frac{\omega\sqrt{2}\pi}{2} \cdot \frac{1}{ch\frac{\omega\pi}{2}};$$

$$\int_0^{+\infty} \cos\omega t\, y_0' dt = -\int_{-\infty}^{0} \cos\omega t\, y_0' dt = -\sqrt{2} + \omega\sqrt{2}\left(-\frac{\pi}{2} th\frac{\omega\pi}{2} + 4\omega \sum_{k=0}^{\infty} \frac{1}{\omega^2 + (1+4k)^2}\right).$$

For the continuation of the local expansion ad infinitum we rebuild it to QPA:

$$y = a_0 + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 + \ldots \to e^{-t} \frac{\alpha_0 + \alpha_1 e^t + \alpha_2 e^{2t}}{1 + \beta_1 e^t + \beta_2 e^{2t}}. \tag{6}$$

For big values of friction we suggest an approach to determine the onset of chaos based on some consequence from the classical Lyapunov stability definition for a case when initial variations are not arbitrary small and limited below. This approach can be considered as an alternative to the well-known Poincaré map approach. Mutual instability of phase trajectories is used as a criterion of chaotic behavior in dynamical systems. One compares trajectories that are initially very close. The proposed stability test shows the mutual stability or instability of the trajectories. Calculations permit to observe a process of appearance and enlargement of the chaotic behavior regions if some selected parameters of the dynamical system under consideration are changing.

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