SUPPRESSING SELF-EXCITED VIBRATIONS IN A COUPLED PENDULUM SYSTEM

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Summary The main objective of this paper is to show the phenomenon of full vibration suppression of a simple two degree of freedom pendulum system by interaction between self-excitation and parametric excitation. Using the averaging method general conditions for full vibration suppression are derived for the linearized system with harmonic stiffness variation. These analytical results are compared with results obtained from a numerical time integration of the linearized and of the original non-linear system.

MECHANICAL SYSTEM

We investigate a mechanical system consisting of two pendulums in a gravity field. Both pendulums are coupled by a linear spring-element. Self-exciting forces are acting on one pendulum and may destabilize the resting coupled pendulum system upon reaching a certain level. The effect of self-excitation is caused by some kind of mechanism that may be represented by an equivalent negative damping coefficient. Well-known examples for such mechanisms are the constant flow around a body with an aerodynamically unfavourable shape [1], or the contact of sliding surfaces with a velocity-dependent friction coefficient [2]. Both mechanisms are being capable of generating self-sustained vibrations if a characteristic parameter (e.g. flow velocity) reaches a critical value. Without counter measures both pendulums would start swinging with increasing amplitudes until nonlinearities in the system cause the vibrations to enter a limit cycle.

Vibration suppression can be achieved by a periodic excitation of the pendulum's pivot in the direction of gravity. It is well known that a single pendulum can be stabilized in its inverted position by this method, which is based on a periodic "modulation" of the gravity forces by the additional inertia forces acting on the pendulum due to the periodic excitation. The combined effect of gravity and inertia results in a periodically changing restoring force, which appears in the equation of motion as a parametric excitation in the displacement-proportional terms. The basic idea of creating a parametric stiffness excitation by a pivot excitation of a pendulum is adopted for the coupled pendulum system. But since we consider a non-inverted 2-dof system under the effect of self-excitation the stabilization of this system is based on a completely different principle, which is applied for the first time to pendulum systems.

For a coupled pendulum system with harmonic parametric excitation the equations of motion can be written as

$$I_{0,1}\ddot{\varphi}_1 + c_1\dot{\varphi}_1 + m_1 g\, l_1 \sin\varphi_1 + k_{12}(\varphi_1 - \varphi_2) = 0,$$
$$I_{0,2}\ddot{\varphi}_2 + c_2\dot{\varphi}_2 + m_2(\ddot{y}_0 + g) l_2 \sin\varphi_2 + k_{12}(\varphi_2 - \varphi_1) = 0,$$
$$\ddot{y}_0 = -\eta^2 A \cos\eta t,$$

and are hence represented by two coupled non-linear differential equations with a time-dependent periodic coefficient. One pendulum is subject to self-excitation (negative damping coefficient c_1) and the second pendulum is parametrically excited (harmonic function $\ddot{y}_0$) as shown in the following Fig. 1.

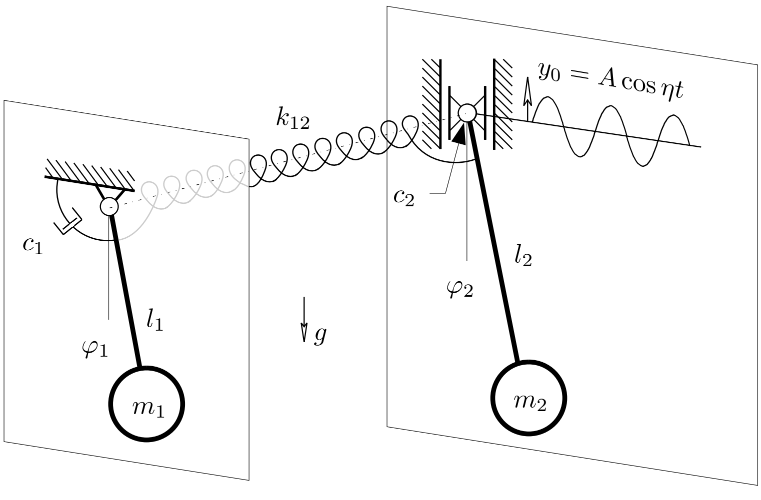

Figure 1. Coupled pendulum system.

AREAS OF VIBRATION SUPPRESSION

To understand the basic behaviour of the original system we first investigate the linearized coupled system for small oscillation amplitudes $\varphi_1,\ \varphi_2$. The equations of motion remain coupled by the constant stiffness coefficient k_{12}. The first equation of motion simplifies to a differential equation with constant coefficients and the second one is equivalent to the well-known Mathieu-equation.

The equations of motion are still quite complicated to be analyzed analytically. Approximate methods are good instruments for understanding the complex behaviour of mechanical systems. It is convenient if the right-hand side of the set of differential equations contains a small parameter. In a first step the system can be solved analytically if this parameter is set to zero. For nonzero but small values of the parameter, solutions can be obtained by perturbation techniques, e.g. the averaging method [3].

We introduce a small parameter for the damping coefficients and the amplitude of pivot excitation (parametric excitation). Transforming the system to its normal form yields the modal frequencies $\Omega_1,\ \Omega_2$. The method of averaging is used to derivate analytically the necessary conditions for full vibration suppression for the linearized system and small excitation amplitudes. These conditions show that the phenomenon of quenching is only possible near specific excitation frequencies of the pendulums pivot, the so-called secondary parametric resonances $\eta_0 = |\Omega_1 - \Omega_2|/N$, where N is a natural number. It is worth to note that the effect of quenching arises not only at a certain frequency value η_0, as it is the case for a dynamic vibration absorbers, but there exists a whole quenching area.

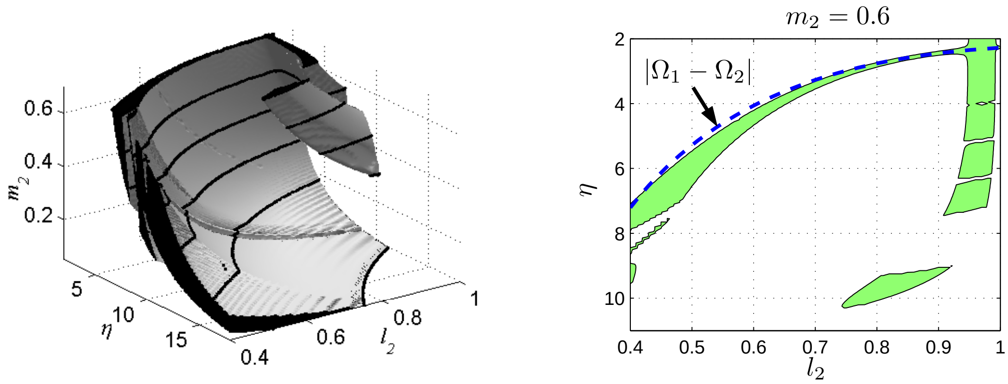

Figure 2. Stability body and chart obtained by numerical integration. Dark area indicates stable system.

The derived analytical conditions for vibration suppression are verified by numerical time integration of the linearized system using Floquet theory. The boundary between stable and unstable motion is investigated for different pivot excitation frequencies η as a function of one or two system parameters and is represented by stability charts and stability bodies, see Fig. 2. The quality of the analytical results for the linearized system are compared numerically with the original non-linear system in the same way.

CONCLUSIONS

In general the self-excited pendulum causes the system to become unstable. Moving the pivot of a second pendulum in the direction of gravity generates a parametric excitation – a harmonic stiffness variation. Coupling both pendulum only by a linear stiffness element can cause the system to become dynamically stable. Although the first pendulum is self-excited which would result in exponentially increasing vibration amplitudes, and also the second pendulum is oscillating due to its pivot motion, the first pendulum is stabilized and gets to rest. This vibration suppression is possible within an area near certain parametric resonance frequencies.

Analytical conditions for vibrations suppression are obtained by applying the averaging method on the linearized system. With these formulas it is possible to predict values of system parameters to achieve the quenching effect.

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