A POWER FLOW MODE THEORY BASED ON INHERENT CHARACTERISTICS OF DAMPING DISTRIBUTIONS IN SYSTEMS AND ITS APPLICATIONS

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POWER FLOW MODE THEORY

Generalised formulation of a dynamical system

For generality, the dynamic equation for a generalized linear system with N degree-of-freedom is represented in the matrix form

$$\mathbf{M}\ \ddot{\tilde{\mathbf{X}}} + \mathbf{C}\dot{\tilde{\mathbf{X}}} + (\mathbf{K} + \mathrm{i}\eta\overline{\mathbf{K}})\tilde{\mathbf{X}} = \tilde{\mathbf{f}}e^{\mathrm{i}\omega t} = \tilde{\mathbf{F}}\,, \tag{1}$$

where $\tilde{\mathbf{F}}$ denotes excitation force vector, $\mathbf{M}$ is a real, symmetric and semi-positive definite mass matrix, $\mathbf{K}$ represents a symmetric and semi-positive definite stiffness matrix, $\mathbf{C}$ is a damping matrix which may be non-symmetrical and $\overline{\mathbf{K}}$ is a real symmetric stiffness matrix relating to hysteretic damping, **,** where η represents a loss factor. For solution $\dot{\tilde{\mathbf{X}}} = \tilde{\mathbf{V}}e^{\mathrm{i}\omega t}$, Eq. (1) reduces to

$$\tilde{\mathbf{f}} = \tilde{\mathbf{Z}}\tilde{\mathbf{V}}\,, \qquad \tilde{\mathbf{Z}} = \mathrm{i}\omega\mathbf{M} + \mathbf{C} + (\mathbf{K} + \mathrm{i}\eta\overline{\mathbf{K}})/\mathrm{i}\omega\,. \tag{2a,b}$$

In general, the impedance $\tilde{\mathbf{Z}}$ is non-symmetrical due to the non-symmetric nature of the damping matrix $\mathbf{C}$.

Power flow mode vector and its characteristic factor

Detailed analysis developed in the paper shows the validity of the result

$$\tilde{\mathbf{V}}^{\mathrm{H}}(\mathbf{C} + \mathbf{C}^{\mathrm{T}} + 2\eta\overline{\mathbf{K}}/\omega)\tilde{\mathbf{V}} = \tilde{\mathbf{V}}^{H}\tilde{\mathbf{f}} + \tilde{\mathbf{f}}^{H}\tilde{\mathbf{V}}\,, \tag{3}$$

where H denotes the Hermitian transpose of the matrix. This allows the time-averaged input power to be expressed as

$$P = \frac{1}{2}\mathrm{Re}\{\tilde{\mathbf{V}}^{\mathrm{H}}\overline{\mathbf{C}}\tilde{\mathbf{V}}\}\,, \tag{4}$$

where, it is proved that, $\overline{\mathbf{C}} = (\mathbf{C} + \mathbf{C}^{\mathrm{T}})/2 + \eta\overline{\mathbf{K}}/\omega$ is a real and symmetric matrix representing the total damping matrix in the system. Based on matrix theory [1], the real symmetric matrix $\overline{\mathbf{C}}$ is decomposed into

$$\overline{\mathbf{C}} = \mathbf{\Phi\Lambda\Phi}^{\mathrm{T}}\,, \tag{5}$$

where $\mathbf{\Lambda}$ is a real diagonal matrix of the eigenvalues λ_j of the matrix $\overline{\mathbf{C}}$ and $\mathbf{\Phi}$ is a corresponding matrix of eigenvectors satisfying $\mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi} = \mathbf{\Phi\Phi}^{\mathrm{T}} = \mathbf{I}$ (a unit matrix). We define λ_j as the *j*th *characteristic damping factor* and eigenvector φ_j as the *j*th *power flow mode vector* of the system. These are linearly independent of each other and they are chosen as a set of base vectors spanning the power flow space and therefore allowe a complete description of the power flow of the system. The velocity vector $\tilde{\mathbf{V}}$ in the power flow space can be decomposed into the form

$$\tilde{\mathbf{V}} = \mathbf{\Phi}\tilde{\mathbf{q}}\,, \qquad \tilde{\mathbf{q}} = \mathbf{\Phi}^{\mathrm{T}}\tilde{\mathbf{V}}\,, \tag{6a,b}$$

where $\tilde{\mathbf{q}}$ is defined as a complex *characteristic velocity* vector. Therefore, the time-averaged power flow expressed by Eq.(4) is now presented as

$$P = \frac{1}{2}\mathrm{Re}\{\tilde{\mathbf{q}}^{\mathrm{H}}\mathbf{\Lambda}\tilde{\mathbf{q}}\} = \frac{1}{2}\sum_{j=1}^{N}\lambda_j\left|\tilde{q}_j\right|^2\,. \tag{7}$$

Eq. (6) provides a *damping-based power mode* expression of the power flow, in which $\lambda_j\left|\tilde{q}_j\right|^2/2$ represents the energy dissipated by the *j*th power flow mode. It is demonstrated by Eq.(7) that the total power can be regarded as the power dissipated by all the N independent power flow modes with each being related to only one characteristic velocity distribution and one characteristic damping factor (i.e., an eigenvalue).

APPLICATIONS OF THE POWER FLOW MODE THEORY

Suspension system with two-degree-of-freedom

Fig.1 illustrates a two mount suspension system which is used to demonstrate application of the generalized power mode theory. Fig. 2 displays the total input power P compared to calculations using other methods [2-4] and the energy dissipated by each power flow mode P_1 and P_2. Fig.3 shows the influence of the system's damping distribution on the total input power spectra.

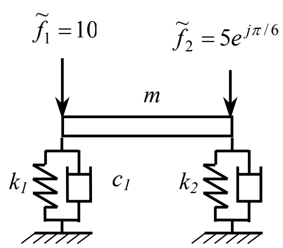

Fig.1 Suspension system

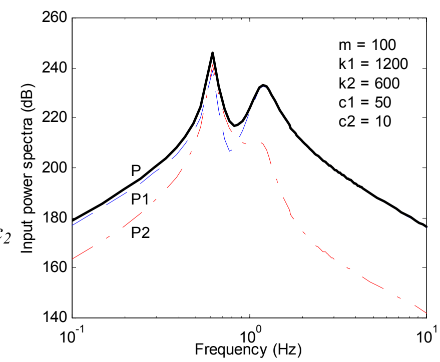

Fig.2 Total power and power flow modes

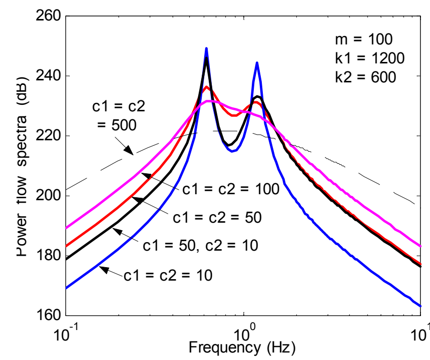

Fig.3 Power affected by system damping

Active vibration control of a human body-seat-boat-wave interaction system

A second example involves a human sitting on a seat mounted by a passive / active suspension system on an elastic vessel travelling in a sinusoidal seaway. In this example, the excitation is a distributed wave load and the system's damping matrix is non-symmetric due to different active damping inputs relating to different control channels [5]. It is demonstrated that the proposed power flow mode approach successfully analyses this complex distributed-lumped system. The effect of feedback control on the input power from waves into the whole coupling system is investigated. More results and detailed analysis are provided in the full paper.

CONCLUSIONS

The developed generalised power flow mode theory demonstrates an effective approach to predict power flow in a dynamical system based on the inherent characteristics of the system's damping distribution. It directly reveals the influence of damping characteristics on energy flow and transmission in the system using its characteristic damping factor and the power flow mode vectors. This power flow mathematical model suggests a design guideline to construct prescribed power flow characteristics through arrangement of the damping distribution in the system.

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