NONLINEAR VIBRATIONS OF GEAR DRIVES

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Summary The contribution presents the modal synthesis method of the mathematical modelling of the gear drive nonlinear vibrations caused by internal excitation generated in gear meshings. Especially undesirable vibrations characterized by discontinuity of mesh gear can be caused by kinematic transmission errors and time dependent meshing stiffnesses in case of small static load. These gear drive impact motions are explained by direct time–integration method and using time series, phase trajectories and Poincaré map.

INTRODUCTION

The presented original modal synthesis method is based on the system decomposition into subsystems, modelling of linearized uncoupled subsystems by FEM, discretization of linear or nonlinear couplings between subsystems, modelling of gyroscopic effects of the rotating subsystems and a assembling of the condensed mathematical model of the system. The condensed mathematical model of the complex coupled system is created by means of spectral ${}^m\boldsymbol{\Lambda}_j$ and modal ${}^m\boldsymbol{V}_j$ submatrices corresponding to the lower master mode shapes of the mutually uncoupled and undamped subsystems (see [1], [2]). The methodology described in this contribution allows to model very complex systems with complicated structure and nonlinear couplings.

MODELLING OF GEAR DRIVES

Let the gear drive (system) be decomposed into rotating shafts with gears (subsystems $j = 1, \dots, N-1$) and a housing (subsystem $j = N$). The shafts are joined together by gear couplings $z = 1, 2, \dots, Z$. The shafts are joined with housing by means the rolling–element bearings. The motion equation of the system, which is decomposed into N subsystems, in the space of their generalized coordinates $\boldsymbol{q}_j(t)$ can be written in the matrix form

$$\boldsymbol{M}_j\ddot{\boldsymbol{q}}_j(t) + (\boldsymbol{B}_j + \omega_j\boldsymbol{G}_j)\dot{\boldsymbol{q}}_j(t) + \boldsymbol{K}_j\boldsymbol{q}_j(t) = \boldsymbol{f}_j^E(t) + \boldsymbol{f}_j^C\,, \quad j = 1, 2, \dots, N\,, \tag{1}$$

where the mass, damping and stiffness matrices $\boldsymbol{M}_j$, $\boldsymbol{B}_j$, $\boldsymbol{K}_j$ of the mutually uncoupled subsystems are symmetric and gyroscopic matrix $\boldsymbol{G}_j$ of the subsystem rotating by angular velocity ω_j ($\omega_N = 0$) is skew–symmetric. The vector $\boldsymbol{f}_j^E(t)$ describes external forced or kinematic excitation. The interaction between the subsystems in the configuration space $\boldsymbol{q}(t) = [\boldsymbol{q}_j(t)]$ of dimension $n = \sum n_j$ can be expressed by global coupling force vector $\boldsymbol{f}_C = [\boldsymbol{f}_j^C]$ in the form

$$\boldsymbol{f}_C(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = -\boldsymbol{B}_B\dot{\boldsymbol{q}}(t) - \boldsymbol{K}_B\boldsymbol{q}(t) + \sum_{z=1}^{Z}\boldsymbol{c}_z F_z(t, \boldsymbol{q}, \dot{\boldsymbol{q}})\,, \tag{2}$$

where $\boldsymbol{B}_B$ and $\boldsymbol{K}_B$ are damping and stiffness matrices of linearized bearing couplings. Vectors $\boldsymbol{c}_z$ (see [1]) and forces $F_z(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ transmitted by gearings correspond to gear coupling z. The elastic parts of these forces are expressed as nonlinear functions of gearing deformations

$$d_z = -\boldsymbol{c}_z^T\boldsymbol{q}(t) + \Delta_z(t)\,, \tag{3}$$

where $\Delta_z(t)$ is the kinematic transmission error on the gear mesh line. In order to make a condensed mathematical model of the system the forces $F_z(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ will be written in the form

$$F_z(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = k_z(t)d_z + b_z\dot{d}_z + f_z(d_z, t)\,, \quad z = 1, 2, \dots, Z\,, \tag{4}$$

where $k_z(t)$ are time dependent meshing stiffnesses, b_z are viscous meshing damping parameters and nonlinear functions $f_z(d_z, t)$ express the influence of an interruption of the mesh gear.

CONDENSED MODEL

The model (1) can be transformed by means of modal submatrices ${}^m\boldsymbol{V}_j \in \mathbb{R}^{n_j, m_j}$ of the uncoupled undamped subsystems into the condensed form using relations $\boldsymbol{q}_j(t) = {}^m\boldsymbol{V}_j\boldsymbol{x}_j(t)$ for $j = 1, 2, \dots, N$ and $m_j < n_j$

$$\begin{aligned}\ddot{\boldsymbol{x}}(t) + (\boldsymbol{D} + \omega_0\boldsymbol{G} + \boldsymbol{V}^T\boldsymbol{B}_B\boldsymbol{V} + \boldsymbol{V}^T\boldsymbol{B}_G\boldsymbol{V})\dot{\boldsymbol{x}}(t) + (\boldsymbol{\Lambda} + \boldsymbol{V}^T\boldsymbol{K}_B\boldsymbol{V} + \boldsymbol{V}^T\boldsymbol{K}_G(t)\boldsymbol{V})\boldsymbol{x}(t) = \\ = \boldsymbol{V}^T[\boldsymbol{f}_E(t) + \boldsymbol{f}_G(t) + \sum_{z=1}^{Z}\boldsymbol{c}_z f_z(d_z, t)]\,.\end{aligned} \tag{5}$$

The new configuration space of the system is expressed by vector $\boldsymbol{x}(t) = [\boldsymbol{x}_j(t)]$ of relatively small dimension $m = \sum m_j$, $m \ll n$. Matrices $\boldsymbol{D} = \mathrm{diag}({}^m\boldsymbol{V}_j{}^T \boldsymbol{B}_j {}^m\boldsymbol{V}_j)$, $\boldsymbol{G} = \mathrm{diag}(\frac{\omega_j}{\omega_0} {}^m\boldsymbol{V}_j{}^T \boldsymbol{G}_j {}^m\boldsymbol{V}_j)$, $\boldsymbol{V} = \mathrm{diag}({}^m\boldsymbol{V}_j)$ are block diagonal and $\boldsymbol{\Lambda} = \mathrm{diag}({}^m\boldsymbol{\Lambda}_j)$ is diagonal matrix composed from the spectral submatrices ${}^m\boldsymbol{\Lambda}_j \in \mathbb{R}^{m_j,m_j}$ of the subsystems. Matrices ${}^m\boldsymbol{V}_j$, ${}^m\boldsymbol{\Lambda}_j$ satisfy the orthonormality conditions ${}^m\boldsymbol{V}_j \boldsymbol{M}_j {}^m\boldsymbol{V}_j = \boldsymbol{I}_j$, ${}^m\boldsymbol{V}_j \boldsymbol{K}_j {}^m\boldsymbol{V}_j = {}^m\boldsymbol{\Lambda}_j$, $j = 1, 2, \ldots, N$.
The interaction between the shafts by means of the gear couplings is expressed by transformed gear coupling matrices $\boldsymbol{K}_G(t) = \sum_{z=1}^{Z} k_z(t) \boldsymbol{c}_z \boldsymbol{c}_z^T$, $\boldsymbol{B}_G = \sum_{z=1}^{Z} b_z \boldsymbol{c}_z \boldsymbol{c}_z^T$, transformed vector of the internal kinematic excitation in gear couplings $\boldsymbol{f}_G(t) = \sum_{z=1}^{Z} [k_z(t)\Delta_z(t) + b_z \dot{\Delta}_z(t)] \boldsymbol{c}_z$ and transformed vector of the nonlinearities in gear couplings $\sum_{z=1}^{Z} \boldsymbol{c}_z f_z(d_z, t)$ in phases of the mesh gear interruption.
The condensed model (5) of order $m = \sum m_j$ can be used for determination of the constant mesh gear regions and simulation of nonlinear system vibration excited by the kinematic transmission errors $\Delta_z(t)$ and time dependent meshing stiffnesses $k_z(t)$, $z = 1, 2, \ldots, Z$. The impact motions are explained using time series, phase trajectories and Poincaré map (see [3]) by direct time–integration of the condensed model. The theory is applied to a simple test–gearbox on Figure 1 (see [4]).

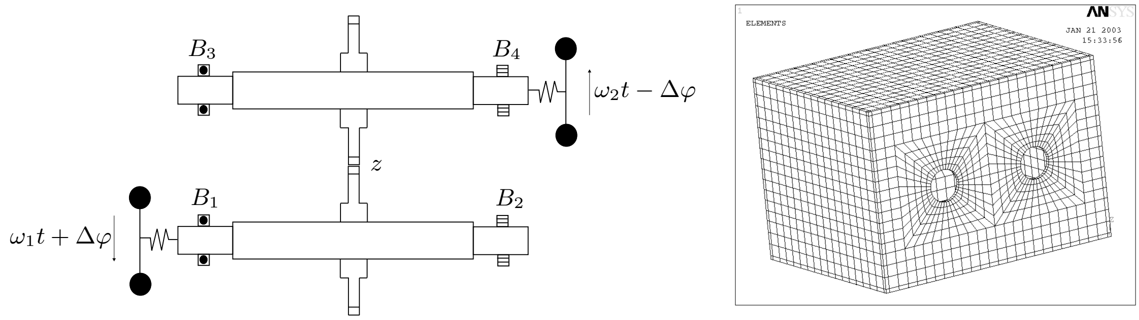

Figure 1. Scheme of the test–gearbox (ω_1 and ω_2 are constant angular speeds)

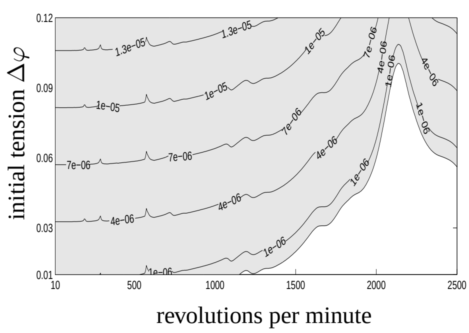

Figure 2. Regions of the constant gear mesh (grey colour) with respect to operating speed of the drive shaft and initial static torsional tension

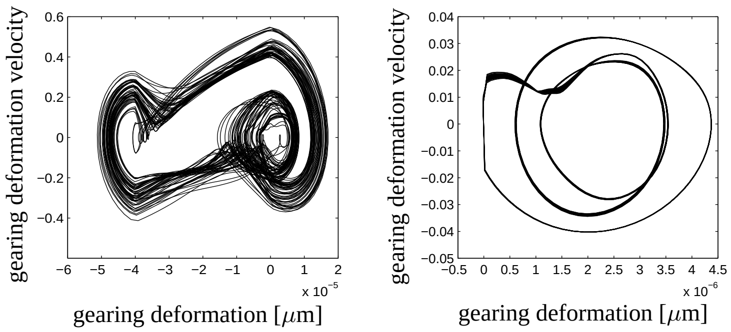

Figure 3. Phase trajectories of the gearing deformation for 1500 rpm of the drive shaft, for two distinct values of the initial tension $\Delta\varphi = 0.015$ rad (left) and $\Delta\varphi = 0.018$ rad (right), and for the tooth backlash $u = 4 \cdot 10^{-5}$ m

CONCLUSIONS

The modal synthesis method for creation of the nonlinear condensed model of gear drives is discussed. This condensed model with small DOF number is constituted by means of the lower undamped vibration mode shapes of the uncoupled subsystems. The maximum and minimum meshing deformations in time and regions of the constant mesh gear are investigated in dependence on the gear drive operating speed and the static load. The condensed model is used for numerical simulation of nonlinear vibration in the phases of the mesh gear interruption caused by the important sources of internal excitation – kinematic transmission errors and time dependent meshing stiffnesses. The impact motions of the gears are explained using time series, phase trajectories and Poincaré map.
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