NON TRIVIAL EFFECT OF STRONG HIGH-FREQUENCY EXCITATION ON A NONLINEAR CONTROLLED SYSTEM

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Summary. Nontrivial biasing effect of high-frequency excitation on a standard nonlinear system with a standard optimal controller is investigated. The interaction between fast excitation and strong damping terms in the control system is identified to be the cause of the permanent control error. A “slow observer” ignoring fast motions is shown to be the simplest way to avoid the undesired bias. All the results are obtained both analytically and numerically.

INVESTIGATED PROBLEM AND OBSERVED EFFECT

Nontrivial effects of high-frequency (HF) excitation on mechanical systems have been investigated intensively in the last decade [1 – 3]. Some of these effects are usually used in controlled systems in form of additional dither signals for example to smoothen out the undesired friction and or to reduce the effective hysteresis. However other effects like stiffening and first of all biasing are normally undesired.

A standard optimal controller for a standard nonlinear system (a movable cart used to balance an inverted pendulum vertically against the acceleration of gravity) is considered in this study. The only difference to the classical control problem [4] is that the system’s base is subjected to synchronized HF-excitation in both vertical and horizontal directions.

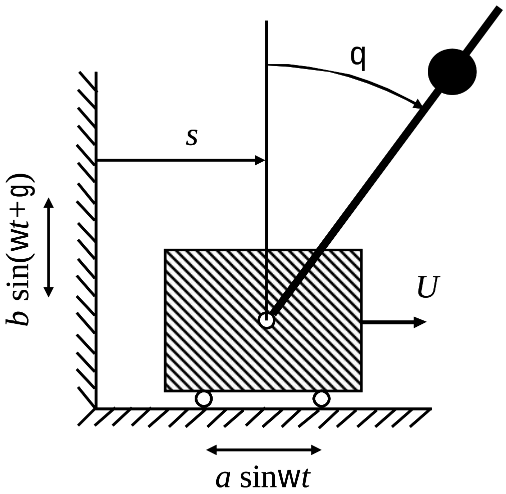

Fig. 1. Inverted pendulum controlled through the moving cart.

This excitation has turned out to affect the behaviour of the considered system dramatically. Both the cart and the pendulum are stabilized on the average by the control system, but the stable position is far away from the desired one even for relative small excitation intensity. Even though the used LQR control is not limited in power and it applies force directly to the cart it is not able to push it in the desired position. The permanent (on the average) control error depends obviously on the intensity of the excitation, but it also depends on the phase difference between vertical and horizontal excitations. The resulting averaged equilibrium can be controlled by the phase difference between the two external HF-excitations. It seems to be even more paradox, that the seemingly obvious idea to increase the control coefficients worsens the situation. On the other hand the whole effect disappears if one of the excitations fails or if their frequencies are different (which would be the case for a standard random noise.

ANALYTICAL PREDICTIONS FOR QUASI-EQUILIBRIUM

The main difference between pure and controlled mechanical systems with respect to HF-vibrations is the damping level. Whereas damping in pure mechanical systems is normally small, its level in the controlled systems is usually high, because usual control strategies try to avoid overshooting. This obstacle makes on the one hand the system unsuitable for the standard averaging; on the other hand it causes the strong interaction between the control system and the external excitation. However a modified averaging procedure suitable for the case of non-critical fast variables [5] can be established and applied in this case. It enables to obtain quite simple approximate expressions for the bias, being quite accurate for the large values of the excitation frequency and sufficiently small values of the average pendulum angle. The approximate solution is given by (1), where $k_1,\ldots,k_4$ are the coefficients of the control system:

$$q_\infty = \frac{\frac{k_2}{w} ab \left(\sin g - \frac{k_2 - k_4}{w} \cos g \right)}{2\left(1+\left(\frac{k_2-k_4}{w}\right)^2\right) - b^2\left(1+\frac{k_2\left(k_2-k_4\right)}{w^2}\right)}; \qquad s_\infty = -\frac{k_3}{k_1} q_\infty \qquad (1)$$

$$U = -k_1 s - k_2 \dot{s} - k_3 q - k_4 \dot{q}$$

This solution shows all the important properties of the effect obtained in numerical simulations:

- The bias exists only in presence of both horizontal and vertical vibrations
- Its average level increases with the excitation intensity, but it also increases with the increased control coefficients (first of all with the coefficient k_2 .
- The bias level depends on and can be controlled by the phase difference between the horizontal and vertical excitations

Analytic prediction for the bias shows, the interaction between fast excitation and strong damping terms in the control system to be the cause of the permanent control error.

From the physical point of view the bias is caused by the asymmetric excitation, leading to an asymmetric averaged stiffness. It moves the initially symmetric equilibrium away from zero, and the pendulum leans slightly, on the average. As a consequence the controller has to stabilize the cart in a position located in the opposite direction to the pendulum lean.

CONCLUSIONS

The described effect does not depend on the specific form of the control algorithm. It seems to be a quite general effect, typical for any kind of system with both external and parametric excitation and sufficiently high damping level. It is also possible to avoid it using quite simple means. The simplest way is to keep the HF-excitation away from the control system. A “slow observer” inserted as a filter between the mechanical system and the signal entrance of the control algorithm accomplishes this task perfectly. The only price, which has to be paid for this solution is the fact, that the “brain” of our system will always see the world through the “slow spectacles”.

References

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