NONLINEAR VIBRATIONS OF JEFFCOTT ROTOR WITH PRELOADED SNUBBER RING

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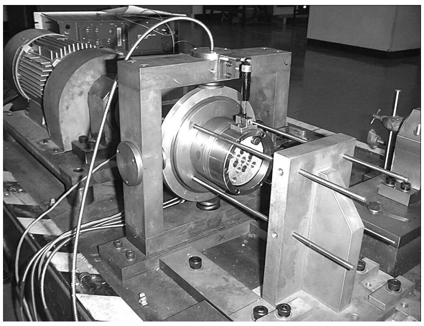
Summary Nonlinear vibrations of a Jeffcott rotor system with a preloaded snubber ring subjected to out of balance excitation are investigated theoretically and experimentally. The details of the design, experimental set-up and mathematical modelling of the system will be presented. The rotor makes intermittent contacts with the preloaded snubber ring and it can produce five different contact regimes which are determined using the principle of the minimum elastic energy in the springs supporting the snubber ring. As a result this rotor system is modelled as a nonlinear piecewise smooth dynamical system, for which a suite of approximate methods has been devised [1]. Chaotic behaviour and co-existence of attractors have been found [2]. A comparison between the theoretical and experimental results made by using bifurcation diagrams, phase portraits and Poincaré maps shows a good correlation between theory and experiments.

JEFFCOTT ROTOR WITH PRELOADED SNUBBER RING

Dynamics of the rotor system with a preloaded snubber ring shown in Figure 1(a) is considered. The experimental rig comprises essentially two main parts, a rigid rotor, which is visco-elastically supported by four flexural rods, and excited by the out-of-balance mass, and a snubber ring, which is also elastically supporting using four compression springs. The rotor assembly consists of a mild steel rotor, running in two angular bearings. Holes are drilled and tapped in both inner sleeves for the addition of imbalance weights. A pair of dampers is attached to the rotor, one in each direction, to provide the system with heavier damping. Four flexural rods are symmetrically clamped at one end to the outer bearing housing and at the other to a large support block. The discontinuous stiffness is provided by a ring to which four compression springs, of much greater stiffness than that of the flexural rods, are symmetrically secured. The rotor runs inside the ring, with a radial clearance between the ring and the outer bearing housing.

(a)



(b)

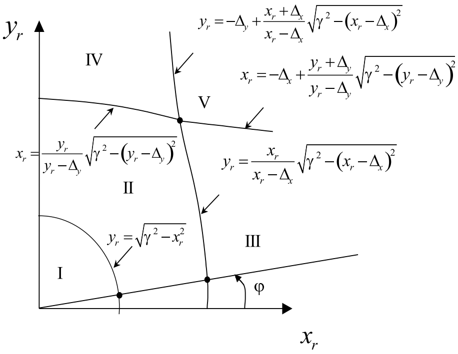


Figure 1. (a) Experimental rig of the investigated rotor system; (b) regions of operation and their boundaries for the first quadrant of (x_r, y_r) plane.

The excitation is provided by an out-of-balance rotating mass $m\rho$. During operation the rotor of mass M makes intermittent contact with the snubber ring. It is assumed that contact is non-impulsive and that the friction between the snubber ring and the rotor is neglected. Also it is assumed that the snubber ring itself is massless. The stiffness and the viscous damping of the snubber ring are equal to k_s and c_s. The stiffness and the damping of the rotor are respectively k_r and c_r. The springs supporting the snubber ring are preloaded by Δ_x in horizontal and Δ_y in vertical directions respectively. There is a gap γ between the rotor and the snubber ring. Also in the initial position, the centre of the rotor is displaced from the centre of the snubber ring by the eccentricity vector $\boldsymbol{\varepsilon}$.

The system can operate in one of two following regimes: (a) no contact and (b) contact between the rotor and the snubber ring [3]. In the latter case, existence of the preloading makes the dynamics of the system more complicated as the force acting from the snubber ring on the rotor depends on whether the displacement of the snubber ring exceeds the preloadings (in one or both directions) or not. Thus the following unique regimes of the system motion can be distinguished: (I) No contact between rotor and snubber ring; (II) Contact between the rotor and the snubber ring, where the both displacements of the snubber ring are smaller than the preloadings, i.e. $|x_s| \leq \Delta_x$ and $|y_s| \leq \Delta_y$; (III) Contact between the rotor and the snubber ring, where the displacement of the snubber ring in the horizontal direction is larger than the preloading, $|x_s| > \Delta_x$, and in the vertical direction is smaller than preloading, $|y_s| \leq \Delta_y$; (IV) Contact between the rotor and the snubber ring, where the displacement of the snubber ring in the horizontal direction is smaller than the preloading, $|x_s| \leq \Delta_x$, and in the vertical direction is larger than preloading, $|y_s| > \Delta_y$; (V) Contact between the rotor and the snubber ring, where the displacements of the snubber ring are larger than the preloadings, i.e. $|x_s| > \Delta_x$ and $|y_s| > \Delta_y$.

MATHEMATICAL MODELLING AND EXPERIMENTAL VERIFICATION

For no contact situation the distance between the centres of the rotor and the snubber ring, $D = \sqrt{(x_r - x_s)^2 + (y_r - y_s)^2}$ is smaller than the gap, γ, that is $D \le \gamma$. Therefore equations of motion for the rotor and the snubber ring are

$$\begin{aligned} &M\ddot{x}_r + c_r\dot{x}_r + k_r(x_r - \varepsilon_x) = m\rho\Omega^2\cos(\varphi_0 + \Omega t), \\ &M\ddot{y}_r + c_r\dot{y}_r + k_r(y_r - \varepsilon_y) = m\rho\Omega^2\sin(\varphi_0 + \Omega t), \\ &c_s\dot{x}_s + k_s x_s = 0, \quad c_s\dot{y}_s + k_s y_s = 0, \end{aligned} \tag{1}$$

where φ_0 is the initial phase shift and Ω is shaft angular velocity.
Once $D = \gamma$, the rotor hits the snubber ring and one or more of the contact regimes may occur, for which the equations of motion can be written as

$$\begin{aligned} &M\ddot{x}_r + c_r\dot{x}_r + k_r(x_r - \varepsilon_x) + F_{s_x} = m\rho\Omega^2\cos(\varphi_0 + \Omega t), \\ &M\ddot{y}_r + c_r\dot{y}_r + k_r(y_r - \varepsilon_y) + F_{s_y} = m\rho\Omega^2\sin(\varphi_0 + \Omega t), \\ &x_s = x_s(x_r, y_r), \quad y_s = y_s(x_r, y_r). \end{aligned} \tag{2}$$

Here the force in the snubber ring $\mathbf{F}_s = (F_{s_x}, F_{s_y})$ varies for different contact regimes. The unknown $x_s(x_r, y_r)$ and $y_s(x_r, y_r)$ give the current location of the snubber ring as a function of the current location of the rotor.
When the rotor and the snubber ring are in contact, the distance between their centres remains constant and equal to the gap, so $(x_r - x_s)^2 + (y_r - y_s)^2 = \gamma^2$. In order to find the location of the snubber ring centre when it moves being in contact with the rotor, the following approach has been adopted.
The problem of finding the current location of the snubber ring has been reduced to finding the minimum of the energy E with the constraint condition $D = \gamma$. This can be done using the Lagrange multipliers method by constructing the Lagrange function $L = E + \lambda\delta$, where λ is Lagrange multiplier, E is the elastic energy of the snubber ring, δ is the constraint function $\delta = (x_r - x_s)^2 + (y_r - y_s)^2 - \gamma^2$. As E and δ are the continuous and differentiable functions, the current position of the snubber ring (x_s and y_s) as a function of the of the current rotor position (x_r and y_r) can be determined from the conditions of the existence of extremum [3]. This allows to analytically describe the location of the snubber ring and to find the borders between different regimes of operation as shown in Figure 1(b).

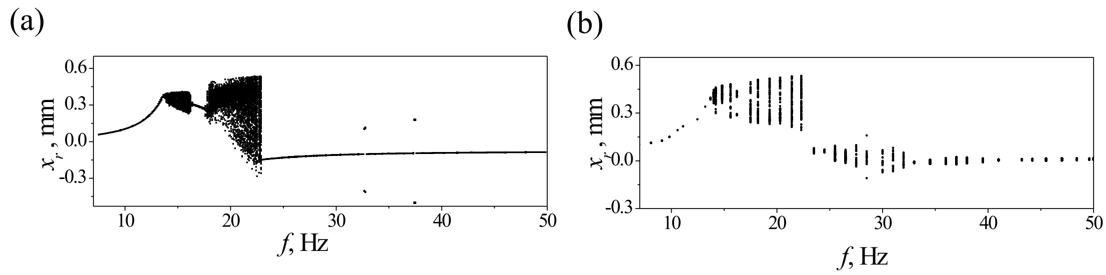


Figure 2. A comparison between (a) theoretical and (b) experimental bifurcation diagrams constructed for varying frequency keeping viscous damping of the snubber ring at $c_s = 3.5\ kg/sec$.

During experiments the response of the rotor system was monitored by noncontacting eddy probes. Two probes were used for the rotor and another two for the snubber ring. Subsequently, the displacement and forcing frequency signals were collected by a data acquisition system LabView, with a purpose-written program controlling rate of sampling, number of samples, calibration and computation of the rotational frequency. The relative velocities of the rotor and the snubber ring $\dot{x}_r$, $\dot{y}_r$, $\dot{x}_s$ and $\dot{y}_s$ were calculated using the LabView digital differentiation facility applied to the output signals from eddy current probes. The data was collated on the computer, where it was scaled, plotted and analysed in the form of Poincaré maps and bifurcation diagrams. A comparison between theoretical and experimental bifurcation diagrams depicted in Figure 2 shows a good correlation [4].

References

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