EXPERIMENTAL STUDY OF NONLINEAR ENERGY PUMPING

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Summary Experimental verification of passive nonlinear energy pumping in a system of coupled oscillators with an essentially nonlinear end attachment is carried out. It is shown that passive transfer of energy (energy pumping) from the linear oscillators to the nonlinear attachment can take place. Agreement between simulated and experimental results was observed, in spite of the strongly nonlinear and transient dynamics of the system considered. The experiments bear out earlier predictions that a significant fraction of the energy introduced directly to a linear structure by an external impulsive (broadband) load can be transferred (pumped) to an essentially nonlinear attachment, and dissipated there locally without spreading back to the system. In addition, the reported experimental results confirm that (a) nonlinear energy pumping occurs above a threshold of the input energy, and (b) resonance capture cascades occur where the nonlinear attachment resonates in sequence with a number of linear modes.

INTRODUCTION

This work provides the first experimental verification of the nonlinear energy pumping phenomenon in a system of two coupled oscillators. The system comprises a damped linear oscillator coupled to a damped nonlinear energy sink (NES) with essential (nonlinearizable) cubic stiffness nonlinearity. Transient (broadband) responses of the system are considered, caused by impulsive excitations. It was shown in earlier works [1,2] that one can separate the dynamics of this system into 'fast' and 'slow' components; in particular, the 'slow' dynamics governs the slowly-varying envelopes (modulation) of the oscillator responses, whereas the 'fast' dynamics describes the fast oscillations that take place inside the envelopes. The fact that the NES interacts with a single linear mode, and that energy pumping takes place in the neighborhood of a 1:1 resonant manifold (i.e.., when the instantaneous frequency of the NES is in the neighborhood of the eigenfrequency of the linear mode), implies that in the experiments considered herein the energy pumping will be dominated by a single fast frequency, namely, one close to the eigenfrequency of the linear subsystem.

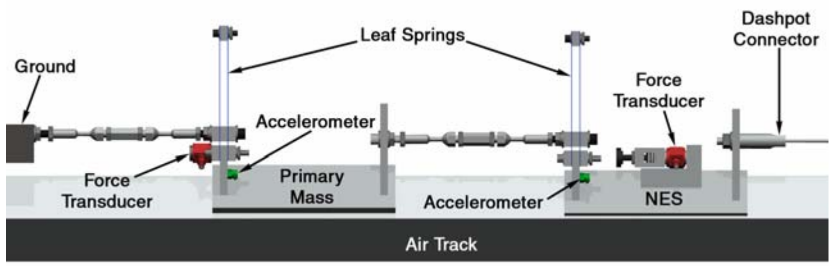


Figure 1. Experimental fixture for nonlinear energy pumping.

EXPERIMENTAL FIXTURE

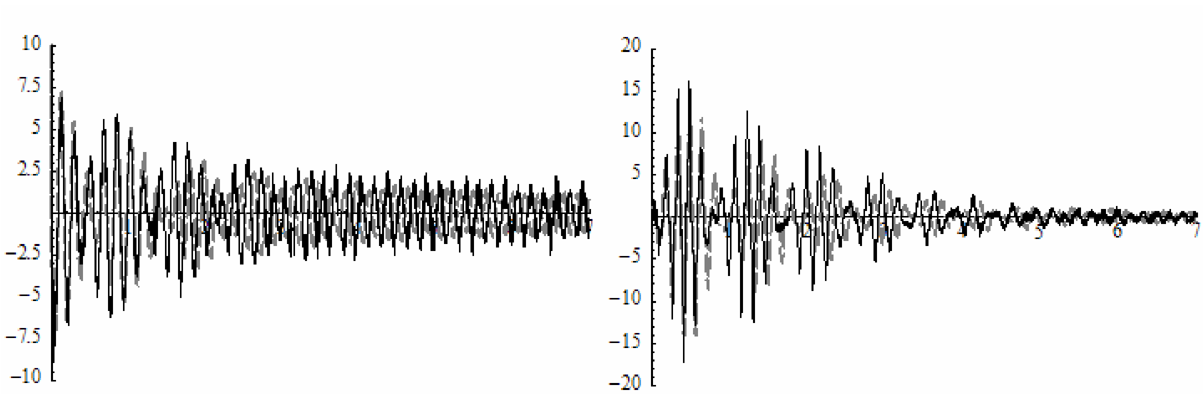
The experimental fixture of Figure 1 consists of two single-degree-of-freedom oscillators (the 'subsystems') connected by means of a linear stiffness. The left oscillator (the linear subsystem) is grounded by means of a linear spring, whereas the right one (the nonlinear energy sink) is grounded by means of a nonlinear spring with essential cubic nonlinearity. To dissipate the pumped energy, a grounded damper exists in the NES. Transient (shock) excitation of the system is provided by means of a rod that impacts elastically with the left mass. The equations of motion are:

$$\begin{gathered} M\ddot{y} + Ky + \varepsilon\lambda\dot{y} + \varepsilon(y - v) = F(t) \\ m\ddot{v} + \varepsilon c\dot{v} + Cv^3 - \varepsilon(y - v) = 0 \end{gathered} \tag{1}$$

with zero initial conditions. Assuming that $F(t)$ is an impulsive (broadband) excitation of finite duration, it is of interest to study the transient (damped and essentially nonlinear) dynamics. Hence, the aim of the experimental work is to show that broadband energy initially imparted to the linear subsystem is passively 'pumped' to the NES where it is confined and dissipates without 'spreading' back to the linear subsystem. Since the essential cubic nonlinearity plays an important role in the realization of the nonlinear energy pumping phenomenon, a discussion of its construction is appropriate. Essential (nonlinearizable) cubic nonlinearity can be achieved by different structural configurations. In this particular experimental fixture, the essential stiffness nonlinearity was realized by adopting the configuration of a thin rod (piano wire) with no pretension clamped at both of its ends, performing transverse vibrations at its center. The geometric nonlinearity of the system considered produces, to the leading order of approximation, an essential cubic stiffness nonlinearity; moreover, the corrective terms as the displacement increases are of higher order and do not produce a linear term in the stiffness characteristic. If, however, the thin wire is preloaded, a highly undesirable linear term, proportional to the initial preload tension, appears, and the resulting stiffness becomes linearizable. Hence, special care is given to minimize pretension in the wire.

RESULTS AND COMPARISONS TO SIMULATIONS

In Figure 2 we present experimental results for a case when an impulsive force of approximately 6.25msec in duration is applied to the linear subsystem with the entire system initially at rest. This forcing level is typical of the strong excitation that is required to induce nonlinear energy pumping in the system under consideration. A comparison between the experimental and theoretical acceleration time series of the two subsystems is depicted, from which very good agreement is noted. Such agreement is typical of what was observed for all the experimental trials. Nonlinear energy pumping is noted, especially at early times of the response (in the period 0 – 4 sec) when the energy of the system is relatively high and the nonlinear effects are more profound. One notes that, in the energy pumping regime 0 – 4 sec*,* the NES oscillates with a dominant 'fast' frequency, which is approximately equal to the eigenfrequency of the linear subsystem. This is a manifestation of resonance capture of the dynamics in the neighborhood of a 1:1 resonance manifold of the system. A quantitative measure of energy pumping in the system is performed by computing the percentage of total input energy that is dissipated at the dashpot of the NES. In this case, eventually, 88.5% of the total input energy is absorbed and dissipated at the NES, a result that demonstrates that the NES is an effective mechanism for passively absorbing and dissipating a significant portion of input energy.



(a) (b)

Figure 2. Nonlinear energy pumping: Experimental ——— and numerical — — — acceleration time series of (a) the linear (directly excited) subsystem and (b) the NES.

CONCLUSIONS

The experiments bear out earlier predictions that a significant fraction of the energy introduced directly to the linear substructure by an external impulsive load can be pumped to the NES and dissipated there locally without spreading back to the linear system. Further trials, for which data are not shown herein, proved passive nonlinear energy pumping in this system to be robust, repeatable, and fully predictable.

Acknowledgments

This work was supported in part by AFOSR Contract 00-AF-B/V-0813 (Dr. Dean Mook, Grant Monitor), and ONR Contract N00014-00-1-0187 (Dr. Louise Couchman, Grant Monitor). Mr. Jeff Kowtko, assisted with the experiments.

References

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