Free Vibration of Sandwich Beams using Timoshenko Theory

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Extended Summary

The free vibration analysis of sandwich beams is a well-researched area and there are literally dozens of published papers on the subject of which references [1 - 6] constitute a small sample. Most of these research papers focus on three-layered sandwich beams and are either based on the classical differential equation approach or on the finite element method, and generally cover a wide range of applications both in terms of boundary conditions and material properties. In this paper, an alternative approach, using symbolic computation is proposed. In particular, the dynamic analysis of a three-layered sandwich beam is carried out by assuming that each component of the beam deforms according to the Timoshenko theory. In order to achieve this, the governing differential equations of motion of the sandwich beam are derived by applying Hamilton's principle and by extensive use of symbolic computation. The equations are solved and boundary conditions are imposed to yield the natural frequencies and mode shapes of some carefully chosen problems. Some details of the theory are given as follows.

Figure 1 shows a three-layered sandwich beam with distinct layers i, where i = 1, 2, and 3, having properties in the usual notation E_i, G_i, A_i, I_i, k_i and ρ_i (i = 1, 2, 3), respectively.

The total potential energy U of the beam due to normal and shearing strains is given by

$$\begin{aligned} U = \frac{1}{2}\int_0^L & \{E_1A_1(u_1')^2 + E_2A_2(u_2')^2 + E_3A_3(u_3')^2 + E_1I_1(\theta_1')^2 + E_2I_2(\theta_2')^2 + E_3I_3(\theta_3')^2\}dx \\ & + \frac{1}{2}\int_0^L \{k_1A_1G_1(w'-\theta_1)^2 + k_2A_2G_2(w'-\theta_2)^2 + k_3A_3G_3(w'-\theta_3)^2\}dx \end{aligned} \tag{1}$$

The total kinetic energy T of the beam considering both axial and rotational velocities can be written as

$$T = \frac{1}{2}\int \{M\dot{w}^2 + \rho_1A_1\dot{u}_1^2 + \rho_2A_2\dot{u}_2^2 + \rho_3A_3\dot{u}_3^2 + \rho_1I_1\dot{\theta}_1^2 + \rho_2I_2\dot{\theta}_2^2 + \rho_3I_3\dot{\theta}_3^2\}dx \tag{2}$$

where the first term is the transverse velocity contribution, and $M = \rho_1A_1 + \rho_2A_2 + \rho_3A_3$.

Applying Hamilton's principle $\delta\int_{t_1}^{t_2}(T-U)dt = 0$, the following governing differential equations are obtained.

$$-E_1A_1u_1'' - \frac{1}{2}E_2A_2u_2'' + \frac{1}{h_2}E_2I_2\theta_2'' + \frac{1}{h_2}k_2A_2G_2(w'-\theta_2) + \rho_1A_1\ddot{u}_1 + \frac{1}{2}\rho_2A_2\ddot{u}_2 - \frac{1}{h_2}\rho_2I_2\ddot{\theta}_2 = 0 \tag{3}$$

$$-\frac{1}{2}E_2A_2u_2'' - E_3A_3u_3'' - \frac{1}{h_2}E_2I_2\theta_2'' - \frac{1}{h_2}k_2A_2G_2(w'-\theta_2) + \frac{1}{2}\rho_2A_2\ddot{u}_2 + \rho_3A_3\ddot{u}_3 + \frac{1}{h_2}\rho_2I_2\ddot{\theta}_2 = 0 \tag{4}$$

$$
\begin{aligned}
&-\frac{h_1}{4}E_2A_2u_2'' - E_1I_1\theta_1'' + \frac{h_1}{2h_2}E_2I_2\theta_2'' - k_1A_1G_1(w'-\theta_1) + \frac{h_1}{2h_2}k_2A_2G_2(w'-\theta_2) \\
&\quad + \frac{h_1}{4}\rho_2A_2\ddot{u}_2 + \rho_1I_1\ddot{\theta}_1 - \frac{h_1}{2h_2}\rho_2I_2\ddot{\theta}_2 = 0
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&\frac{h_3}{4}E_2A_2u_2'' + \frac{h_3}{2h_2}E_2I_2\theta_2'' - E_3I_3\theta_3'' + \frac{h_3}{2h_2}k_2A_2G_2(w'-\theta_2) - k_3A_3G_3(w'-\theta_3) \\
&\quad - \frac{h_3}{4}\rho_2A_2\ddot{u}_2 - \frac{h_3}{2h_2}\rho_2I_2\ddot{\theta}_2 + \rho_3I_3\ddot{\theta}_3 = 0
\end{aligned} \tag{6}
$$

$$
M\ddot{w} - k_1A_1G_1(w''-\theta_1') - k_2A_2G_2(w''-\theta_2') - k_3A_3G_3(w''-\theta_3') = 0 \tag{7}
$$

The above five equations form the fundamental basis of this paper. The complete analysis, which provides the solution of these equations and numerical simulation of a number of representative problems, will be reported in the full-length paper. The investigation is currently in progress. The significance of the results and a detailed parametric study will also be reported in the full-length paper.

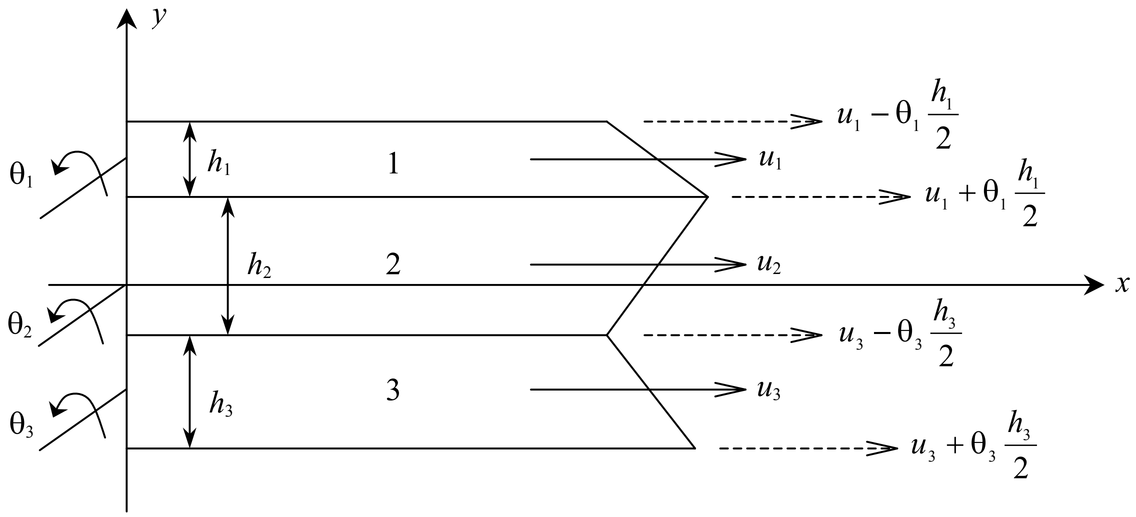

Figure 1. Co-ordinate system and notation for a three-layered sandwich beam

References

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