OPTIMIZATION OF FUNCTIONALLY GRADED MATERIALS WITH TEMPERATURE DEPENDENT PROPERTIES. A MESHFREE SOLUTION

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Summary We find optimal material composition for FGM by selecting points on a spline-interpolated volume fraction curve as material design variables. For the local effective properties of the FGM we use the rule of mixtures and the Mori-Tanaka model. We minimize the mass while stresses are used as constraints. The nonlinear thermo-elastic problems are solved using a meshfree method. The design space of material configurations is unrestricted and we study convergence properties when the number of design variables increases.

INTRODUCTION

Inspired by some naturally occurring material systems such as bamboo, bone, sea-shells, teeth, functionally graded materials (FGMs) are heterogenous materials featuring a smooth gradation for achieving a specific function. Most commonly, metal-ceramic FGMs are used to make the transition from a metal part which is strong but cannot operate at high temperatures and a ceramic part that is efficient in shielding against high temperatures but has low tensile strength. Some current and potential applications include thermal protective shields of high reliability, thin films and coatings for cutting tools, dental and prosthetic implants, to name just a few.

The function of the smooth gradation characteristic to FGMs is to reduce or eliminate stress concentrations and/or jumps in bimaterial systems, minimize damage due to thermal shock or mechanical impact, etc. Some of the previous theoretical results have analyzed the behavior of FGMs when the function that defines volume fraction variation is given by a monotonic, power-law type function (x^p, $p \in \mathbb{R}$). The monotonic power-law is selected only as a matter of convenience, analytical results are difficult, if not impossible, to obtain otherwise. The finite element method and meshfree methods have also been employed in the study of FGMs (see e.g. [1], [2]). Determining the optimal volume fraction variation for extremizing a specific objective function has been discussed in, for example, [3]. In [3], the function describing the volume fraction is limited to piecewise quadratic. In addition, the temperature dependence of the material properties is ignored.

In this paper we analyze optimal configurations for FGMs in a more general setting: the material variation we use is a shape-preserving spline that interpolates points which are the material design variables. By changing the shape of the volume fraction function we modify the material composition of the FGM. We take into account the temperature dependence of material properties and use a meshfree solution for the thermoelastic problem for possible extensions to combined shape-material optimization of FGMs. The meshfree methods offer considerable benefits when applied to shape optimization problems as recently shown in [4], [5].

PROBLEM FORMULATION

We consider a domain Ω that is occupied by the FGM under thermomechanical loading. The nonlinear heat-transfer equations and the equations of the thermoelastic equilibrium can be written as:

$$\left\{ \begin{array}{lll} \kappa(\boldsymbol{x},\theta)\Delta\theta + Q(\boldsymbol{x},\theta) = 0 & \text{in} & \Omega \\ \theta = \theta_0 & \text{on} & \Gamma_\theta^0 \\ \kappa(\boldsymbol{x},\theta)\nabla\theta \cdot \boldsymbol{n} = \overline{q} & \text{on} & \Gamma_\theta^1 \\ \kappa(\boldsymbol{x},\theta)\nabla\theta \cdot \boldsymbol{n} + h(\boldsymbol{x},\theta)\ (\theta - \theta_\infty) = 0 & \text{on} & \Gamma_\theta^2 \end{array} \right. , \quad \text{nonlinear heat-transfer} \tag{1}$$

$$\left\{ \begin{array}{lll} \nabla \cdot \boldsymbol{\sigma}(\boldsymbol{x},\theta) + \boldsymbol{b}(\boldsymbol{x},\theta) = \boldsymbol{0} & \text{in} & \Omega \\ \boldsymbol{\sigma}(\boldsymbol{x},\theta)\ \boldsymbol{n} = \overline{\boldsymbol{t}}(\boldsymbol{x},\theta) & \text{on} & \Gamma_{\boldsymbol{t}} \\ \boldsymbol{u}(\boldsymbol{x},\theta) = \overline{\boldsymbol{u}}(\boldsymbol{x},\theta) & \text{on} & \Gamma_{\boldsymbol{u}} \end{array} \right. , \ \text{where} \boldsymbol{\sigma} = \mathcal{C}(\boldsymbol{x},\theta) : [\boldsymbol{\varepsilon}(\boldsymbol{x},\theta) - \alpha(\boldsymbol{x},\theta)\ \theta(\boldsymbol{x})\mathbf{I}] \tag{2}$$

The corresponding weak-forms for these equations are obtained using the element-free Galerkin (EFG) meshfree method. The variation of the material is captured at the level of the Gauss integration points which are *independent* of the discretization nodes. This is one advantage of the meshfree solution when compared to a FEM solution.

The material parameters of the FGM depend not only on the volume fraction of the two components, but also on the temperatures. The temperature dependence of the thermal expansion coefficients for the materials uses in this paper (ZrO_2 and Ti-6Al-4V) is shown in Figure 1. The critical tensile strength variation is given in Figure 2. The compressive critical strength, thermal conductivity, and Young's modulus, also vary with the temperature.

At a location where the volume fraction of metal is $v(x)$, we evaluate the "composite" material parameters via a rule-of-mixture (ROM) scheme. Other schemes can be selected and easily implemented. We use the ROM for lack of experimental data. The same ROM scheme is used for estimating the critical tensile and compressive stresses. For example, let $S^{metal}(\theta)$ and $S^{ceramic}(\theta)$ be the critical tensile strengths for the metal and the ceramic materials at a given temperature θ. The estimated critical tensile strength at a location where the volume fraction of the metal is $v(x)$ will then be given by:$S(x,\theta) = S^{metal}(\theta) \cdot v(x) + S^{ceramic}(\theta) \cdot [1 - v(x)]$.

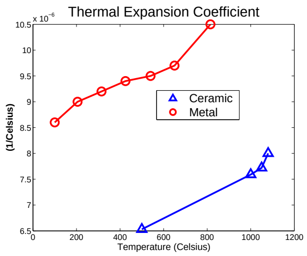


Figure 1. Temperature variation of the thermal expansion coeff.

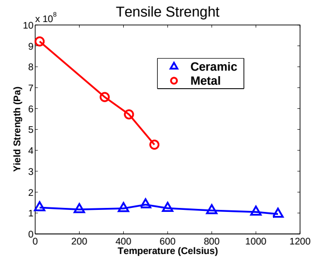
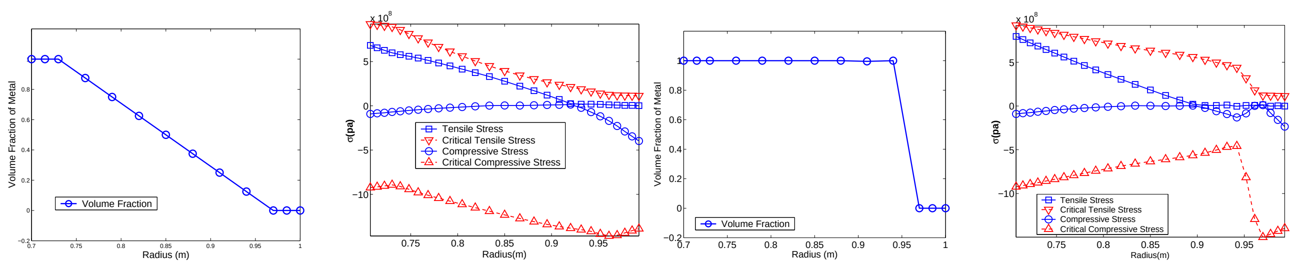


Figure 2. Temperature dependence of the tensile strength

OPTIMIZATION AND NUMERICAL RESULTS

Using the EFG method, we discretize the weak forms of the thermal and thermomechanical response. The heat-transfer problem is *nonlinear*, but the nonlinearity is weak. We use a fixed-point iteration and obtain convergence in just a couple of iterations. With temperature field obtained in this fashion, we calculate the displacements, strains, and stresses. The solution of the thermoelastic problem is repeated at every iteration of the optimization algorithm. We consider a *quarter of a hollow cylinder* with an inner radius of 0.7m and outer radius of 1m. After a convergence analysis, we use 2349 discretization points and 2240 integration cells. Each cell uses 5by5 Gaussian integration. We impose inner pressure 100MPa and the inner temperature is 20 C° while the outer temperature reaches 800C°.

The results for one example are given below. We *minimize the mass* while a constraint is defined in terms of violating the critical tensile and compressive stresses. This leads to a constrained *nonlinear optimization problem* that we solve using SQP. We start with a configuration that has thin coatings of pure metal and pure ceramic at the exterior with a thick FGM layer in-between. We use nine design. We converge to a profile that has a thin FGM layer. The metal (ZrO_2) is lighter than the ceramic (Ti-6Al-4V) and the resulting FGM is metal rich.



(a) Initial vol. frac. (b) Initial stress profile (c) Optimal vol. frac. (d) Final stress profile

Figure 3. The initial and final volume fraction functions and the corresponding critical stress profiles.

CONCLUSIONS

Optimal profiles in minimizing mass under stress constraints for temperature dependent FGMs have been obtained. The points on the material variation function are the design variables. The meshfree solution leads to fast convergence of the SQP optimization algorithm used here. Future plans include combining shape and material optimization for non-symmetrical loading.

References

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