DESIGN OF ARTICULATED MECHANISMS WITH A DEGREE OF FREEDOM CONSTRAINT USING GLOBAL OPTIMIZATION

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Summary This paper deals with design of articulated mechanisms using a truss ground structure representation. The considered mechanism design problem is to maximize the output displacement for a given input force by choosing a prescribed number of truss elements out of all the available elements, so that the resultant mechanism has one mechanical degree of freedom when supported in a statically determinate manner. The mechanical degree of freedom constraint is included since it is essential for obtaining a proper articulated mechanism design. The Green-Lagrange strain measure is used to accommodate for large displacements. The problem is formulated as a non-convex mixed integer problem and solved using a convergent deterministic global optimization method based on branch and bound with convex relaxations.

INTRODUCTION

Research on design of compliant mechanisms using topology optimization techniques in continuum structures[1] suggests that it should also be possible to obtain results for articulated[1] mechanisms by applying topology optimization. For design of articulated mechanisms the concept of *mechanical* degrees of freedom (DOF)[2] becomes critical for obtaining a proper mechanism design, while it is an insignificant feature for compliant mechanism design as such mechanisms do not have real joints.

It seems natural and advantageous to represent articulated mechanisms by truss elements and pin joints. Firstly, one can directly interpret the truss topology from the kinematic diagram. Also, it is possible to accommodate large displacements (geometrical non-linearity) without element distortion problems. Finally, with this interpretation one can formulate an optimization problem in terms of the so-called ground structure approach for truss topology design.

The idea of the truss ground structure approach is to first give a large set of potential elements in a design domain, then eliminate unnecessary elements, or equivalently chose the necessary elements, by solving an optimization problem. Finally, the optimal solution defines the resultant mechanism design. One truss ground structure is shown in Fig. 1.

The following general assumptions on the geometrical non-linearity are made. Large displacements are allowed but the strains are assumed to be small. The strain is given by the Green-Lagrange strain measure and the material properties are assumed to be linear elastic.

Based on the combinatorial truss ground structure approach with the geometrical non-linearity and the mechanical DOF constraint taken into account an optimization problem with objective to maximize the displacement at a certain point of the structure is stated. The problem is formulated as a non-convex mixed integer optimization problem. One can tackle this problem, for instance, in a nested formulation using gradient-based optimization techniques together with a penalization of non-integer values of the design variables. However, one cannot expect to find a global optimal solution because of the non-convexity. Furthermore, it may be very difficult even to find a feasible solution in many situations.

In this paper we therefore propose to solve the difficult non-convex combinatorial problem using a deterministic global optimization technique based on branch and bound[4]. The proposed branch and bound method can implicitly examine all possible topologies. This allows for identifying the global optimal solution for the first time in this area of application.

PROBLEM FORMULATION

One relevant mechanism design problem is to maximize the output displacement for a given input force by picking N truss elements out of the n possible elements, so that the resultant mechanism has $f_{\mathrm{DOF}}(x) = 1$ degree of freedom when supported in a statically determinate manner. The problem can be formulated as a non-convex mixed integer optimization problem in the variables $(x, f, \epsilon, u) \in \mathbb{B}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^d$, where x is the truss connectivity vector; f is the normal force vector; ϵ is the strain vector; u is the nodal displacement vector; n is the number of design variables; d is the number of displacement variables. The problem format is thus:

$$
\begin{array}{lll}
\underset{x,f,\epsilon,u}{\text{maximize}} & q^\top u (= D_{\mathrm{out}}) & \text{(Output displacement)} \\
\text{subject to} & R(f,u) = 0, & \text{(Equilibrium equations)} \\
& f_j = x_j E_j \epsilon_j, & \text{(Force-strain relationship)} \\
& \epsilon_j = b_j^\top u + \dfrac{1}{2} u^\top B_j u, & \text{(Green-Lagrange strain measure)} \\
& f_{DOF}(x) = 1, & \text{(Mechanical degree of freedom)} \\
& e^\top x = N, & \text{(Element number constraint)} \\
& u^{\min} \le u \le u^{\max}, & \text{(Displacement bounds)} \\
& x \in \{0,1\}^n, & \text{(Binary design variables)}
\end{array}
$$

[1]From Oxford Advanced Learner's Dictionary: Articulate (*technical*), to be joined to something else by a joint, so that movement is possible.

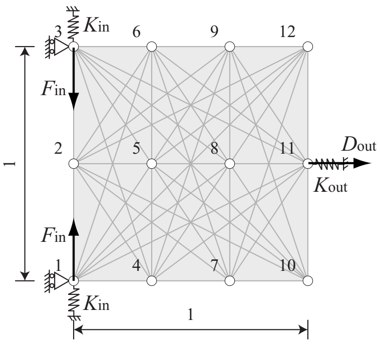

Figure 1. Truss ground structure.

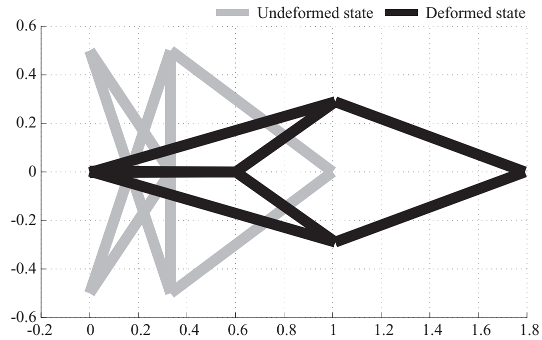

Figure 2. Optimal eight-bar scissors-like mechanism.

where $j = 1, \ldots, n$; $q \in \mathbb{R}^d$ is a given vector defining the displacement of the output port; $N > 0$ is a given integer limiting the number of bars in the optimal structure; $f_{\text{DOF}}(x)$ denotes the mechanical degrees of freedom; $e \in \mathbb{R}^n$ is a vector of all ones; $b_j \in \mathbb{R}^d$ contains the direction cosines divided by the length of the j-th bar; the symmetric positive semidefinite matrix $B_j \in \mathbb{R}^{d \times d}$ defines the nonlinear part of the strain measure; $u^{\min} \in \mathbb{R}^d$ and $u^{\max} \in \mathbb{R}^d$ are given lower and upper bounds on the displacements. The reason for including force variables is that they decrease the nonlinearity in the equilibrium equations. Furthermore, the introduction of the force variables gives the opportunity to enforce stress constraints in a straight-forward way. This problem may also be extended to multiple loads and path-generation problems.

OUTLINE OF THE BRANCH AND BOUND METHOD

In order to perform global optimization on the above problem we have to overcome several obstacles in order to define a convex relaxation. Firstly, the design variables are binary; secondly, the forces f_j are defined with bilinear equality constraints; thirdly, the strains ϵ_j are defined with quadratic equality constraints; fourthly, the equilibrium equations $R(f, u) = 0$ are bilinear; finally, the constraint on the mechanical degree of freedom is non-convex and only well-defined for designs with binary values ($x \in \{0, 1\}^n$).

In the proposed branch and bound method the binary constraints on the design variables and the mechanical degrees of freedom constraint are relaxed and subsequently dealt with in the branch and bound tree. The bilinear equality constraints defining the forces are equivalently written as a set of linear constraints utilizing the fact that the bilinear term contains binary design variables. The remaining bilinear and non-convex quadratic constraints are relaxed using convex envelopes. The relaxation becomes a convex problem with a linear objective function and linear and convex quadratic constraints which can be efficiently solved using sequential quadratic programming. Branching is first performed on the design variables and then on the displacement variables.

NUMERICAL EXAMPLE

To illustrate the capabilities of the method a 66-bar test example is solved. The test example that we employ is shown in Fig. 1. The figure shows the truss ground structure of possible nodes and connections as well as all necessary boundary conditions, load cases and other related parameters. The springs and forces at the input ports model a strain based input actuator (e.g., shape memory alloy or piezoelectric device) and the output spring models a resistance to the movement.

We have applied the proposed global optimization method to the above problem in the case of $V = 8$ bars. The global optimal solution is known for this case from a graph-theoretical enumeration method[3]. This provides a validation of the method proposed in this paper. We have successfully obtained the optimal design which is shown in Fig. 2. Other examples have also been treated successfully.

References

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