DENSITY GRADIENT BASED REGULARIZATION OF TOPOLOGY OPTIMIZATION PROBLEMS

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Summary The numerical solution of topology design problems often results in optimal designs containing microstructures characterized by oscillating material density fields such as the well-known "checkerboard-patterns", whose realization turns out to be difficult from the engineering point of view. In this context we introduce a density gradient based regularization approach including a global penalty functional, which prevents the formation of oscillating density distributions. Furthermore we discuss numerical aspects of the proposed regularization and present solutions of the regularized topology optimization problem for exemplary design problems.

FORMULATION & REGULARIZATION OF THE TOPOLOGY OPTIMIZATION PROBLEM

The formulation of topology design problems requires in general the introduction of a discrete material-indicator function $\rho : \mathbf{x} \rightarrow \{0, 1\}$ as design variable [1], which divides the design domain Ω into a solid region $\Omega^s = \{\mathbf{x} \in \Omega \mid \rho(\mathbf{x}) = 1\}$ and an empty region $\Omega^e = \{\mathbf{x} \in \Omega \mid \rho(\mathbf{x}) = 0\}$. To obtain a continuous optimization problem, instead of a large-scale combinatorial problem, the indicator function is usually identified as the material density and intermediate values between 0 and 1 are admitted. In addition special penalty methods such as the SIMP-approach [1] are used to reduce the set of admissible solutions to so-called "black&white" designs free of "gray" regions characterized by intermediate density values, whose realization turns out to be difficult from the engineering point of view. In the case of linear-elastic maximum-stiffness-design the corresponding optimization problem can be formulated in the following form [1]

$$\min_{\rho} \left\{ \int_{\Omega} \mathbf{f}_v^T \mathbf{u} \, d\Omega + \int_{\Gamma_t} \mathbf{f}_t^T \mathbf{u} \, d\Gamma_t \right\}$$

$$\text{subject to :} \tag{1}$$

$$\int_{\Omega} \boldsymbol{\varepsilon}^T(\mathbf{u}) \, \mathbf{C}(E) \, \boldsymbol{\varepsilon}(\delta\mathbf{u}) \, d\Omega = \int_{\Omega} \mathbf{f}_v^T \delta\mathbf{u} \, d\Omega + \int_{\Gamma_t} \mathbf{f}_t^T \delta\mathbf{u} \, d\Gamma_t \ , \ E = \rho^p E_0 \quad , \quad p > 1$$

$$\int_{\Omega} \rho \, d\Omega \le M_0 \ , \ 0 \le \rho \le 1.$$

The displacement field $\mathbf{u}$ caused by body forces $\mathbf{f}_v$ and traction forces $\mathbf{f}_t$, acting on the boundary Γ_t of the design domain, is determined by the variational form of the equilibrium condition, which can be handled as a constraint to the optimization problem. In this context $\mathbf{C}$ denotes the material rigidity tensor and E corresponds to the Young's modulus of the material. Nevertheless the implementation of the SIMP-approach or equivalent penalty methods results in discontinuities in the global density distribution and often leads to designs containing unfavourable microstructures such as the well-known "checkerboard patterns" [3] characterized by oscillations of the density function between the discrete values 0 and 1.
To obtain "black&white" designs free of microstructures the optimization problem (1) can be regularized by the so-called X-SIMP-approach [2]

$$E = \rho^p E_0 e^{-\gamma\lambda} \quad , \quad p > 1 \ , \ \gamma > 0 \ , \ \lambda = \int_{\Omega} (\nabla\rho)^T \, \nabla\rho \, d\Omega, \tag{2}$$

where γ corresponds to an additional penalty parameter and λ represents a global penalty functional based on the gradient of the density function, which prevents the formation of oscillating density distributions characterized by high density gradients. In consideration of the above penalty-approach the topology optimization problem (1) has to be modified as follows

$$
\begin{gathered}
\min_{\rho} \left\{ \int_{\Omega} \mathbf{f}_v^T \mathbf{u} \, d\Omega + \int_{\Gamma_t} \mathbf{f}_t^T \mathbf{u} \, d\Gamma_t \right\} \\
\text{subject to :} \\
\int_{\Omega} \boldsymbol{\varepsilon}^T(\mathbf{u}) \, \mathbf{C}(E) \, \boldsymbol{\varepsilon}(\delta \mathbf{u}) \, d\Omega = \int_{\Omega} \mathbf{f}_v^T \delta \mathbf{u} \, d\Omega + \int_{\Gamma_t} \mathbf{f}_t^T \delta \mathbf{u} \, d\Gamma_t \\
E = \rho^p E_0 e^{-\gamma \lambda} \quad , \quad p > 1 \quad , \quad \gamma > 1 \quad , \quad \lambda = \int_{\Omega} (\nabla \rho)^T (\nabla \rho) \, d\Omega \\
\int_{\Omega} \rho \, d\Omega \leq M_0 \ , \ \ 0 \leq \rho \leq 1.
\end{gathered}
\tag{3}
$$

In this context we present results of numerical studies of different maximum-stiffness-design problems, such as the examples stated below (figure 1), and we compare optimal designs obtained on the basis of the SIMP-approach and the X-SIMP-regularization. Furthermore we discuss numerical aspects of the proposed regularization approach such as the finite-element-discretization of the regularized problem, the formulation of an appropriate finite-element-type and the corresponding stiffness-matrix, the sensitivity analysis and the numerical solution of the discretized optimization problem.

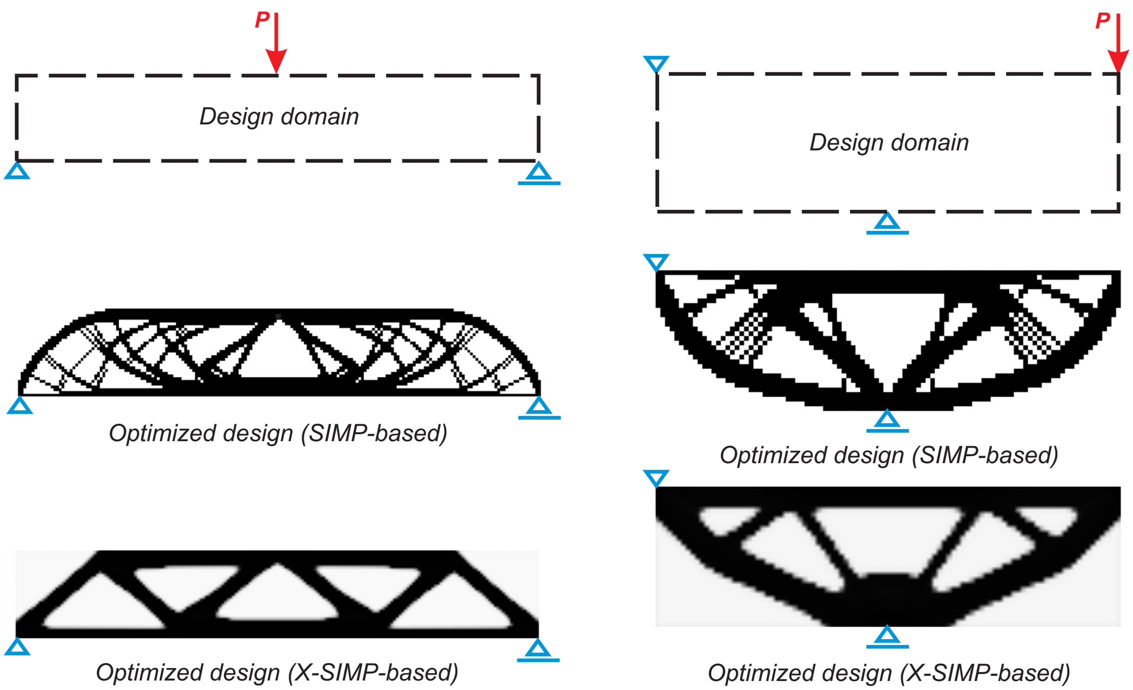

Figure 1. Topology design based on the SIMP-approach and the X-SIMP-regularization

References

[1] Bendsøe, M.P.: Optimal shape design as a material distribution problem. Struct. Optim. **1**, 193-202, 1989

[2] Kotucha, G., Hackl. K.: Density gradient enhanced topology optimization of continuum structures. Proc. Appl. Math. Mech. **3**, 292-293, 2003

[3] Sigmund, O.; Petersson, J.: Numerical instabilities in topology optimization: a survey on procedures dealing with checkerboards, mesh-dependencies and local minima. Struct. Optim. **1**6, 68-75, 1998