Coupled sensitivity analysis and design optimization for thermo-structural systems*

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Summary Thermo-structural systems coupled disciplines of heat conduction and structural mechanics are widely used in engineering. The paper presents a systematical methodology for the design optimization of such systems using the coupled sensitivity analysis and mathematical programming methods on structural behaviors of thermal stresses and thermal buckling.

INTRODUCTION

Thermo-structural systems abound in many industrial engineering such as aerospace and aeronautical engineering, thermodynamic machinery, electronic devices and material engineering. The thermal responses of such system[1], e.g. thermal displacements, stresses, and buckling are commonly the predominant factors in the design phase. Therefore a comprehensive design of thermo-structural system should take the heat conduction and the structural mechanics into account simultaneously. However in some former researches, the designers had to adopt some simplified methods when conducting the optimal design of such system in order to reduce the problem difficulties. For example, the common way was to neglect the coupled effects of the two disciplines in the phases of sensitivity analysis and design optimization. In fact, the rationality of the simplified methods is questionable.

Over the past two decades, the sensitivity analysis and design optimization for structural system have gained fruitful achievements[2], which also placed a solid foundation and offered possible ways for design optimization of thermo-structural system. This investigation makes an attempt to extend the mature techniques to the design optimization of thermo-structural systems, based on which, further attentions may be paid to the design optimization for other kinds of structural multidisciplinary systems such as electro-structural, magneto-structural, acoustic-structural systems.

Two kinds of sensitivity analysis problems are considered in the present research using the direct and the adjoint methods. One is the coupled sensitivity analysis and design optimization of thermal stresses problem. In this respect, many contributions, Ref[3] and references therein, have been published. But most of their attentions were paid to the deduction of sensitivity equations by employing the continuum model (based on the partial differential equations) and neglected the discussions of solution methods. This paper present a set of clearer and more concise deductions for the sensitivity analysis based on the finite element equations, which are suitable for the computer implementations. The other is the coupled sensitivity analysis of thermal buckling problem. There are limited references in this field. Ref[4] discussed the uncoupled thermal buckling problem for the laminated composites. Ref[5] considered the coupled effects but didn't offer procedures on the sensitivity analysis. We present the coupled sensitivity equations in detail for the thermal buckling problem.

Based on the results of sensitivity analysis, the mathematical optimization model is constructed and solved by SLP/SQP algorithms. The computer implementations are performed on the software platform-JIFEX[6], which incorporates size design variables, shape design variables, the technique of semi-analytical sensitivity analysis and the geometrical modeling techniques. The implemented software system can be applied to the general purposes design optimization for the plane, axi-symmetric, and complex thin-walled thermo-structural problems. Numerical examples are employed to validate the present methods and reveal the necessity of coupled sensitivity analysis.

COUPLED SENSITIVITY ANALYSIS OF THERMAL STRESS

Static problem

In the static problem, the finite element equations of heat conduction and structural mechanics are expressed as

$$\boldsymbol{K}\boldsymbol{T} = \boldsymbol{R} \tag{1}$$

$$\boldsymbol{K}^{\mathrm{m}}\boldsymbol{u} = \boldsymbol{f} \tag{2}$$

Where $\boldsymbol{K}$,$\boldsymbol{T}$,$\boldsymbol{R}$ are the heat conductance matrix, the nodal temperature vector and the heat load vector, respectively. And $\boldsymbol{K}^{\mathrm{m}}$, $\boldsymbol{u}$, $\boldsymbol{f}$ are the structural stiffness matrix, the nodal displacement vector, and the external load vector. Suppose that in the optimal model, the index function (objective or constraint function) is $g(x, \boldsymbol{T}, \boldsymbol{u})$ in which x is the design variable. Here just one design variable is considered for simplicity, the same formulations can be extended to multiple design variables problems. Then the sensitivity of g is

$$\mathrm{d}g/\mathrm{d}x = \partial g/\partial x + (\partial g/\partial \boldsymbol{T})(\mathrm{d}\boldsymbol{T}/\mathrm{d}x) + (\partial g/\partial \boldsymbol{u})(\mathrm{d}\boldsymbol{u}/\mathrm{d}x) \tag{3}$$

In sensitivity analysis, the direct method is to derive the Eqn(1) and (2) with respect to x and to apply the results into Eqn(3) to compute the sensitivity of index function. The adjoint method needs to introduce two adjoin vectors $\boldsymbol{\Lambda}_{\mathrm{h}}$ and $\boldsymbol{\Lambda}_{\mathrm{m}}$ which satisfy the following aojoint equations:

$$\boldsymbol{K}^{\mathrm{m}}\boldsymbol{\Lambda}_{\mathrm{m}} = (\partial g/\partial \boldsymbol{u})^{\mathrm{T}} \text{ and } \boldsymbol{K}\boldsymbol{\Lambda}_{\mathrm{h}} = (\partial g/\partial \boldsymbol{T})^{\mathrm{T}} + (\mathrm{d}\boldsymbol{f}/\mathrm{d}\boldsymbol{T})^{\mathrm{T}}\boldsymbol{\Lambda}_{\mathrm{m}} \tag{4}$$

Then the sensitivity of the index function g is

$$\mathrm{d}g/\mathrm{d}x = \partial g/\partial x + \boldsymbol{\Lambda}_{\mathrm{h}}^{\mathrm{T}}(\mathrm{d}\boldsymbol{R}/\mathrm{d}x - (\mathrm{d}\boldsymbol{K}/\mathrm{d}x)\boldsymbol{T}) + \boldsymbol{\Lambda}_{\mathrm{m}}^{\mathrm{T}}(\partial \boldsymbol{f}/\partial x - (\mathrm{d}\boldsymbol{K}^{\mathrm{m}}/\mathrm{d}x)\boldsymbol{u}) \tag{5}$$

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Quasi-static problem

In the quasi-static problem, the structural mechanics equations are Eqn(2) and the heat conduction is time dependent.

$$\boldsymbol{M}\dot{\boldsymbol{T}} + \boldsymbol{K}\boldsymbol{T} = \boldsymbol{R} \tag{6}$$

Where $\boldsymbol{M}$ is the heat capacity matrix. The index function is defined as

$$g(x,\boldsymbol{T},\boldsymbol{u}) = \int_0^{t_f} p(x,\boldsymbol{T},\boldsymbol{u},t)\mathrm{d}t \text{ and } \mathrm{d}g/\mathrm{d}x = \int_0^{t_f} \left(\partial p/\partial x + (\partial p/\partial \boldsymbol{T})(\mathrm{d}\boldsymbol{T}/\mathrm{d}x) + (\partial p/\partial \boldsymbol{u})(\mathrm{d}\boldsymbol{u}/\mathrm{d}x)\right)\mathrm{d}t \tag{7}$$

Then the direct method is to derive Eqn(6) and (7) with respect to x and to apply the results into Eqn(7) to get dg/dx. With the introducing the adjoint vectors $\boldsymbol{\Lambda}_\mathrm{h}$ and $\boldsymbol{\Lambda}_\mathrm{m}$ which satisfy

$$\boldsymbol{K}^\mathrm{m}\boldsymbol{\Lambda}_\mathrm{m} = (\partial p/\partial \boldsymbol{u})^\mathrm{T} \text{ and } -\boldsymbol{M}\dot{\boldsymbol{\Lambda}}_\mathrm{h} + \boldsymbol{K}\boldsymbol{\Lambda}_\mathrm{h} = (\partial p/\partial \boldsymbol{T})^\mathrm{T} + (\partial \boldsymbol{f}/\partial \boldsymbol{T})^\mathrm{T}\boldsymbol{\Lambda}_\mathrm{m} \tag{8}$$

The adjoint method gives

$$\mathrm{d}g/\mathrm{d}x = \int_0^{t_f} \left(\partial p/\partial x + \boldsymbol{\Lambda}_\mathrm{h}^\mathrm{T}\left(\mathrm{d}\boldsymbol{R}/\mathrm{d}x - (\mathrm{d}\boldsymbol{M}/\mathrm{d}x)\dot{\boldsymbol{T}} - (\mathrm{d}\boldsymbol{K}/\mathrm{d}x)\boldsymbol{T}\right) + \boldsymbol{\Lambda}_\mathrm{m}^\mathrm{T}\left(\partial \boldsymbol{f}/\partial x - (\mathrm{d}\boldsymbol{K}^\mathrm{m}/\mathrm{d}x)\boldsymbol{u}\right)\right)\mathrm{d}t \tag{9}$$

Obviously, the above formulations are clearer and more concise than those based on continuum model[3].

COUPLED SENSITIVITY ANALYSIS OF THERMAL BUCKLING

Based on the results of heat conduction Eqn(1) and structural analysis Eqn(2), the stuctural buckling equations are

$$\left(\boldsymbol{K}^\mathrm{m} + \lambda \boldsymbol{K}_\sigma^\mathrm{m}\right)\boldsymbol{\varphi} = 0 \tag{10}$$

Where $\boldsymbol{K}_\sigma^\mathrm{m}$, $\boldsymbol{\varphi}$, and λ are the geometrical stiffness matrix, the buckling mode vector and the critical load coefficient, respectively. Then in the sensitivity analysis, the direct method gives

$$\mathrm{d}\lambda/\mathrm{d}x = -\boldsymbol{\varphi}^\mathrm{T}\left(\mathrm{d}\boldsymbol{K}^\mathrm{m}/\mathrm{d}x + \lambda\left(\mathrm{d}\boldsymbol{K}_\sigma^\mathrm{m}/\mathrm{d}x\right)\right)\boldsymbol{\varphi} \tag{11}$$

Prior to the solution of Eqn(11), the sensitivities of Eqn(1) and (2) should be obtained to compute the sensitivity for $\boldsymbol{K}_\sigma^\mathrm{m}$ because it is a function of $\boldsymbol{x}$, $\boldsymbol{T}$ and $\boldsymbol{u}$. After some manipulations, the adjoint method gives

$$\frac{\mathrm{d}\lambda}{\mathrm{d}x} = -\boldsymbol{\varphi}^\mathrm{T}\left(\partial \boldsymbol{K}^\mathrm{m}/\partial x - \lambda\left(\partial \boldsymbol{K}_\sigma^\mathrm{m}/\partial x\right)\right)\boldsymbol{\varphi} - \boldsymbol{\Lambda}_\mathrm{m}^\mathrm{T}\left(\partial \boldsymbol{f}/\partial x - (\mathrm{d}\boldsymbol{K}^\mathrm{m}/\mathrm{d}x)\boldsymbol{u}\right) - \boldsymbol{\Lambda}_\mathrm{h}^\mathrm{T}\left(\mathrm{d}\boldsymbol{R}/\mathrm{d}x - (\mathrm{d}\boldsymbol{K}/\mathrm{d}x)\boldsymbol{T}\right) \tag{12}$$

Where the adjoint vectors $\boldsymbol{\Lambda}_\mathrm{h}$ and $\boldsymbol{\Lambda}_\mathrm{m}$ satisfy the adjoint equations

$$\boldsymbol{K}^\mathrm{m}\boldsymbol{\Lambda}_\mathrm{m} = \boldsymbol{w}^\mathrm{T}, \left(w_i = \lambda\boldsymbol{\varphi}^\mathrm{T}\left(\partial \boldsymbol{K}_\sigma^\mathrm{m}/\partial u_i\right)\boldsymbol{\varphi}\right) \text{ and } \boldsymbol{K}\boldsymbol{\Lambda}_\mathrm{h} = (\partial \boldsymbol{f}/\partial \boldsymbol{T})^\mathrm{T}\boldsymbol{\Lambda}_\mathrm{m} \tag{13}$$

DESIGN OPTIMIZATION AND SOFTWARE IMPLEMENTATION

Based on the sensitivity results, the optimal model can be solved by mathematical programming methods: SLP/SQP. The above sensitivity analysis methods are implemented in the software JIFEX[6] of structural design optimization which incorporates semi-analytical method and geometrical modeling technique for all kinds of size and shape design variables. Hence this work can be applied directly to industrial thermo-structural problems with general purposes.

CONCLUSIONS

The main attentions are paid to the systematically coupled sensitivity analysis of thermo-structural system, including the thermal stress and thermal buckling problems. The work reveals the necessity of coupled sensitivity analysis in this respect, and places a solid foundation for the future work on other similar coupled design optimization of structural systems.

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