SHAPE OPTIMIZATION OF THERMOMECHANICAL STRUCTURES IN THE PRESENCE OF CONVECTION AND RADIATION USING PARALLEL EVOLUTIONARY COMPUTATION

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Summary Shape optimization of heat conducting, elastic bodies subjected to thermal and mechanical loads is considered. Interaction of stress and temperature fields is modeled using the boundary element formulation of steady state thermoelasticity. The presence of heat radiation with mutual irradiation of the boundaries and the presence of shadow zones is taken into account. A parallel evolutionary algorithm is used to evaluate the optimal shape.

INTRODUCTION

The physical problem addressed in the paper is evolutionary shape optimization of thermoelastic bodies whose boundaries exchange heat by convection and radiation and mechanical loads. The novel aspect in the study is the presence of mutual irradiation of concave portions of the boundary. Such model contains strongly nonlinear behavior due to the fourth power Stefan Boltzmann law. Another difficulty is caused by the presence of the shadow zones. The presence of heat radiation in the mathematical models of heat transfer is often neglected. Inclusion of radiation in the model becomes crucial at elevated temperatures, where this heat transfer mode dominates and may be responsible for more that 90% of the total energy transport.

In the shape optimization problems the domains and their boundaries are changing during the iteration process. Every evaluation of the objective function requires a solution of a discretized boundary value problem preceded by a generation of a new numerical grid. Because the boundary element method (BEM) is typically limited only to the boundary its application in shape optimization arises in a natural way.

Evolutionary algorithms (EA) mimic the evolution of subsequent generations of living systems. This class of algorithms does not require evaluation of sensitivity coefficients and is treated as the global optimization technique. The main and very serious drawback of these techniques is their high computational cost. This is partially mitigated by invoking the parallel computations.

FORMULATION OF THE BOUNDARY VALUE PROBLEM

An elastic body occupying a domain Ω bounded by a boundary Γ is considered. The physical problem is considered in the terms of the BEM approach.

Conduction

The heat conduction equation converted into an equivalent integral equation takes the form

$$c(\mathbf{r})T(\mathbf{r}) = \int_{\Gamma}\left[T^*(\mathbf{r},\mathbf{p})q(\mathbf{p}) - q^*(\mathbf{r},\mathbf{p})T(\mathbf{p})\right]d\Gamma(\mathbf{p}) \tag{1}$$

where T and q are the temperature and the heat flux fields on the boundary, respectively, T^* and q^* are temperature and heat flux fundamental solutions, respectively, at points **r** generated by a unit pointwise heat sources located at point **p** of an infinite medium

Radiation

Radiation is governed by an integral equation linking the radiative heat flux q^r and the blackbody emissive power $e_b=\sigma T^4$, where σ is the Stefan Boltzmann constant. For an enclosure bounded by a closed surface Γ_c the equation takes the form

$$q^r(\mathbf{r}) + \varepsilon(\mathbf{r})e_b(\mathbf{r}) = \varepsilon(\mathbf{r})\int_{\Gamma_c}\left[e_b(\mathbf{p}) + \frac{1-\varepsilon(\mathbf{p})}{\varepsilon(\mathbf{p})}q^r(\mathbf{p})\right]\frac{\cos\phi_r\cos\phi_p}{2|\mathbf{r}-\mathbf{p}|}\beta(\mathbf{r},\mathbf{p})d\Gamma_c(\mathbf{p}) \tag{2}$$

where ε is the emissivity, ϕ_r and ϕ_p are angles defined by line connecting points **r** and **p** and normal at these points, β is the Boolean function.

Thermoelasticity

Boundary integral equations for thermoelasticity are given in the form

$$c_{ij}(\mathbf{r})u_j(\mathbf{r}) + \int_{\Gamma}\left[p_{ij}^*(\mathbf{r},\mathbf{p})u_j(\mathbf{p}) - u_{ij}^*(\mathbf{r},\mathbf{p})p_j(\mathbf{p})\right]d\Gamma(\mathbf{p}) + \int_{\Gamma}\left[P_i(\mathbf{r},\mathbf{p})T(\mathbf{p}) - Q_i(\mathbf{r},\mathbf{p})q(\mathbf{p})\right]d\Gamma(\mathbf{p}) \tag{3}$$

where u_{ij}^* and p_{ij}^* are fundamental solutions of elastostatics, P_i and Q_i are known functions.

Eqs (1), (2) and (3) are completed by suitable boundary conditions. For every geometrical configuration the conjugate problem of heat and stress analysis is solved using BEM. The strongly nonlinear heat transfer problem modeled by equation of conduction (1) and radiation (2) is solved with respect to the boundary temperatures, conductive $\mathbf{q}^c$ and radiative $\mathbf{q}^r$ heat fluxes. The coupling between these equations is due to the continuity of temperature and the total heat balance on the radiating surface $\mathbf{q}=\mathbf{q}^c + \mathbf{q}^r$. Heaving boundary temperatures and fluxes the displacement and traction fields are evaluated solving (3).

EVOLUTIONARY OPTIMIZATION

There are many possible objective functions that may be defined for shape optimization of the considered problem. In the paper two types of optimization criteria were examined: (Case I) minimum volume of the structure (Eq. 4) and (Case II) maximum amount of heat dissipated from a portion of the boundary Γ_d (Eq. 5)

$$\min_{ch} ff_I; \quad where \quad ff_I = \int_{\Omega} d\Omega \qquad (4) \qquad \text{and} \qquad \max_{ch} ff_{II}; \quad where \quad ff_{II} = \int_{\Gamma_d} q d\Gamma_d \qquad (5)$$

with constraints imposed on admissible temperature and equivalent stress fields (Case I) and on the admissible volume of the structure (Case II)

A chromosome *ch* contains genes that are represented by real numbers and play the role of design variables describing the shape of the structure. The parallel EA performs evolutionary process in the same manner as their sequential counterparts. The difference is in the fitness evaluation. While for the sequential process all members of the population are processed by the same CPU, the values of the fitness function are computed concurrently by M processors. The approach used in this study was to allot to one processor the task of computing the fitness function corresponding to one chromosome. The parallel AE has a master-slave architecture. Using a random process the *master* unit generates the initial population consisting of chromosomes whose genes assume admissible values. These chromosomes are transferred to the *slave* units. In the next step slaves transmit the values of the fitness function to the master. Communication between master and slaves is accomplished invoking the socket mechanism. The generation of a new population of chromosomes is carried out by the master by executing appropriate evolutionary operators (rank selection, cloning, simple crossover and Gauss mutation). The resulting set of new chromosomes is transmitted to the slaves where the fitness function is evaluated. The process of generation of new populations is terminated when the stop criterion is fulfilled.

NUMERICAL EXAMPLES

Numerical tests were performed for shape optimization of a heat radiator used to dissipate heat from electrical devices (Fig. 1). The problem was solved for both criteria (4) and (5) aking into account the symmetry of the structure. In both cases 14 design variables described the shape. The parameters of AE used in this study were: 20 chromosomes, 14 genes and 200 generations. The best radiator optimal shapes are shown in Fig. 2a (Case I) and in Fig. 2b (Case II).

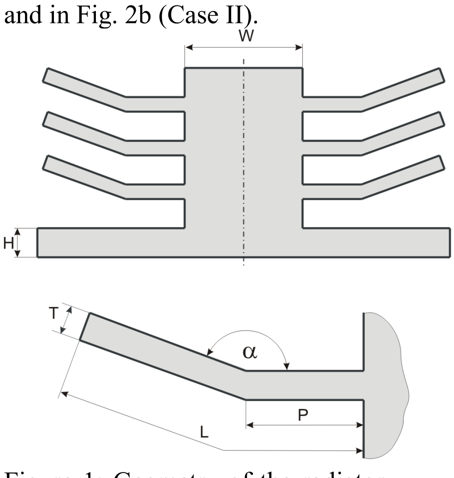

Figure 1: Geometry of the radiator and design variables

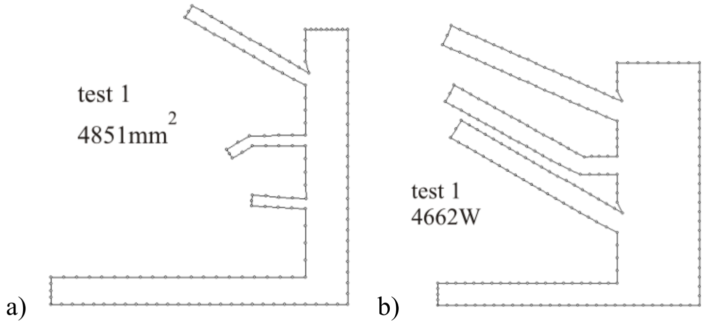

Figure 2: Optimal solutions, a) case I, b) case II

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