TOPOLOGY OPTIMIZATION OF VIBRATING STRUCTURES WITH HYDRODYNAMIC SURFACE PRESSURE LOADING

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Summary This paper deals with the problem of topology optimization of continuum structures subjected to time-harmonic hydrodynamic surface pressure loading. This problem is an extension relative to traditional topology optimization, as both the locations and directions of the loading change as the structural topology changes. Maximization of the integral dynamic structural stiffness for a given value of the frequency of the hydrodynamic pressure loading is the design objective, and the total structural volume is prescribed.

INTRODUCTION

In the design of structures and machines against vibration and noise, many problems are concerned with structures subjected to surface pressure loading like hydrodynamic pressure loading that varies harmonically in time. An important feature of this class of problems is that the surface loads are not only dynamic, but also depend on the design itself, i.e. both the locations and directions of the loads will change with changes of the structural topology and shape. As a few examples we may mention hydrodynamic pressure loading in pump housings and pressure containers, and loading due to fluctuating internal fluid flow in ducts, diffusers and turbomachinery.

In this paper, maximum integral dynamic structural stiffness is chosen as the design objective for prescribed hydrodynamic pressure amplitude and frequency. The boundary conditions, the material, and the total volume of material within the admissible design domain are also given. The volumetric densities of material in the finite elements in the design domain are the design variables. The SIMP material model [1] is used. The design dependence of the loading implies two principal extensions relative to usual topology design problems. Firstly, the sensitivity analysis of the objective function also involves the sensitivities of the loads with respect to the design variables. Secondly, an enhanced design model must be considered, that encounters one or more parameterized, smooth surface domains associated with a prescribed iso-volumetric density of the material. These surface domains serve to define the action of the design dependent loading on the structure, and are determined by the design variables. This was first described by Hammer and Olhoff [2]. In their method, the design dependent loads are assumed to be applied on the surface of the potential structural domain, which is generated by iso-volumetric points of material density during the topology evolution. This method was developed further by Du and Olhoff [3] to automatically handle possible problems of "separate isolines" and "isoline islands" that may appear in the generation of the loading curve (for 2D problems), and the method was extended to 3D problems.

FORMULATION OF THE TOPOLOGY OPTIMIZATION PROBLEM

The problem of optimizing the topology of a structure for maximum value of the integral dynamics structural stiffness is equivalent to minimizing the integral dynamic compliance, which can be formulated as follows:

$$\begin{aligned} &\min_{\rho_e} \{C_d = \mathbf{P}^{\mathrm{T}}\mathbf{U}\} \\ &Subject\ \ to: \\ &\quad (-\overline{\omega}^2\mathbf{M} + \mathbf{K})\mathbf{U} = \mathbf{P}\ , \\ &\quad \sum_{e=1}^{N_E} \rho_e V_e - V^* \le 0\ , \quad (V^* = \alpha V_0)\ , \\ &\quad 0 < \underline{\rho} \le \rho_e \le 1\ , \quad (e = 1, \cdots, N_e)\ . \end{aligned} \tag{1}$$

The dynamic compliance is equal to the work done by the external dynamic forces against corresponding displacements, and is proportional to the expression given for C_d in (1), where $\mathbf{U}$ denotes the displacement amplitude vector of the steady-state vibration, and $\mathbf{P}$ denotes the design-dependent load magnitude vector of the external surface loading. The harmonic external loading vector $\mathbf{p}(t)$ with the given excitation frequency $\overline{\omega}$ can then be expressed as $\mathbf{p}(t)=\mathbf{P}e^{i\overline{\omega}t}$ and the displacement response vector as $\mathbf{a}(t)=\mathbf{U}e^{i\overline{\omega}t}$. The symbols $\mathbf{K}$ and $\mathbf{M}$ represent the N dimensional structural stiffness and mass matrices, where N is the number of DOFs. The symbol ρ_e is the volumetric material density of element e and is the design variable in the problem. The symbol α denotes the volume fraction of available material and is given by V^*/V_0, where V_0 is the volume of the admissible design domain and V^* is the prescribed total volume of available material. The symbol N_e represents the total number of finite elements.

Sensitivity analysis

The sensitivity of the objective function C_d with respect to the design variables ρ_e is given by

$$C_d' = (\mathbf{P}^{\mathrm{T}}\mathbf{U})' = \mathbf{P}'^{\mathrm{T}}\mathbf{U} + \mathbf{P}^{\mathrm{T}}\mathbf{U}'\ , \tag{2}$$

where prime denotes partial derivative with respect to ρ_e. The sensitivity $\mathbf{P}'$ of the design-dependent load can be calculated by finite differences. The sensitivity $\mathbf{U}'$ of the displacement is given by

$$(\mathbf{K}-\varpi^2\mathbf{M})\,\mathbf{U}' = \mathbf{f} \equiv \mathbf{P}'-(\mathbf{K}'-\varpi^2\mathbf{M}')\,\mathbf{U}\ , \tag{3}$$

where the sensitivities of the stiffness matrix and the mass matrix can be directly obtained from the SIMP material model. Optimization problem (1) can be solved by using the well-known MMA method or an optimality criteria method.
During the design process, the method presented in Pedersen [4] is used to suppress the influence of localized eigenmodes on the global structural response.

RESULTS AND DISCUSSION

Not surprisingly, maximization of the integral dynamic stiffness normally yields a structure whose eigenfrequencies are far from the prescribed excitation frequency ϖ of the hydrodynamic load; this structural behaviour implies efficient avoidance of resonance phenomena with large displacement amplitudes and low dynamic stiffness. As is well known from other problems of dynamic design of engineering structures, a difficulty that may need to be considered, is how to maintain the *static* stiffness (corresponds to $\varpi = 0$) of the structure when optimizing its *dynamic* properties. In the present topology optimization problem, the initial design, which is normally represented by a uniform distribution of material with intermediate density over the admissible design domain, may have a low fundamental eigenfrequency Ω in comparison with the given loading frequency ϖ. In this case, an increase of the dynamic stiffness corresponding to ϖ normally implies a decrease of the static stiffness due to a decrease of the fundamental eigenfrequency Ω (thereby avoiding resonance). As a result of this, the structure may become very weak at the (local) optimum of the dynamic stiffness that is obtained. In order to prevent this, one may introduce a lower bound constraint on the static stiffness. However, this will delimit the gain of the optimization of the dynamic stiffness. In fact, much larger values of the dynamic stiffness can be obtained if we can start out the optimization procedure using a value of ϖ that is lower than the value of Ω for the current design, and then sequentially increase ϖ up to its originally prescribed value. This procedure has the desirable effect of generating a series of topologies with increasing values of both Ω and the static and dynamic stiffnesses for the sequence of structures produced. Finally, the procedure delivers the optimum dynamic stiffness topology solution subject to the originally prescribed loading frequency ϖ. The procedure automatically avoids resonance, and works very well as long as the prescribed loading frequency ϖ is lower than the optimum value Ω_{opt} of the fundamental eigenfrequency, i.e. $\varpi < \Omega_{opt}$.
Now, if the prescribed value of ϖ is such that $\varpi > \Omega_{opt}$, the optimization of the dynamic stiffness will drive the fundamental eigenfrequency Ω towards zero. At the same time, the static displacements of the structure become very large, which means that the static stiffness tends to zero. The physical reason for this behaviour is that, in the limit, a disintegration is created in the structure. In this limit, the zero value of the fundamental eigenfrequency is associated with a rigid body vibration mode, and the static displacements of the disintegrated part of the structure become infinite, as the structure cannot sustain the static load.
A straight-forward way of avoiding this unwanted structural behaviour is to include a lower bound constraint on the static stiffness in the mathematical formulation of the problem of maximizing the dynamic stiffness. We have found that such a constraint is extremely effective and well-chosen when optimizing the dynamic stiffness for a value of $\varpi > \Omega_{opt}$.

NUMERICAL EXAMPLE

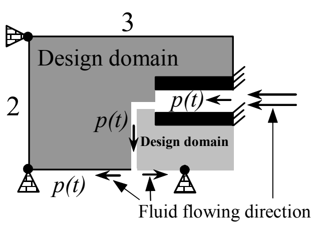

Fig. 1. Initial design

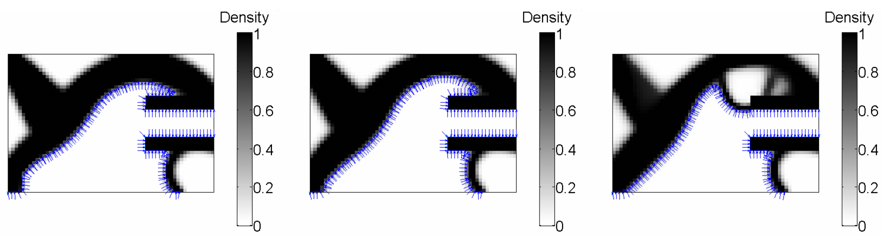

Fig. 2. (a) $\varpi = 0$ (b) $\varpi = 800$ (c) $\varpi = 1000$

This example shows optimum topology designs of an inlet. The fluid flows in the channel of the inlet as shown in Fig. 1, and exerts a hydrodynamic pressure loading on the inner wall of the inlet. Figs. 2(a-c) show the optimized topologies (volume fraction 40%) for the three loading frequencies $\varpi = 0$ (static loading), $\varpi = 800$ and $\varpi = 1000$.

References

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