SHAPE SENSITIVITY ANALYSIS FOR FIXED-GRID ANALYSIS BASED ON OBLIQUE BOUNDARY CURVE APPROXIMATION

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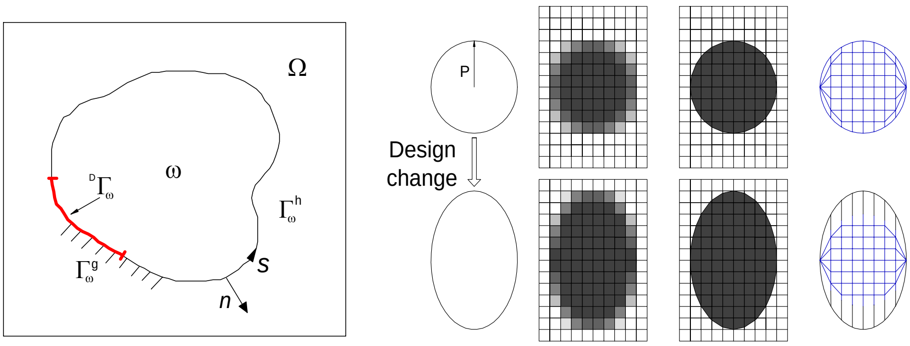
Summary Fixed-grid analyses equipped with a fictitious domain method can avoid remeshing for shape optimization, but stresses on domain boundaries cannot be calculated accurately if the boundaries are improperly represented. For improved stress evaluation, we consider the direct boundary curve approximation by piecewise oblique lines which can cross boundary elements. In this approach, the intersection points between the fixed girds and the approximated boundary do not necessarily coincide with the analysis nodes unlike in existing fixed-grid analyses. The objective of this investigation is to derive the analytic shape sensitivity of stresses for the direct piecewise oblique boundary approximation. Since the force term in the sensitivity equation is associated only with the elements crossed by the design boundary curve, only the design velocities of the intersection points between the curve and the fixed mesh are needed for sensitivity analysis.

INTRODUCTION

By the standard shape optimization based on the finite element approach, remeshing cannot be avoided during the optimization process if accurate analysis is required, especially for design problems requiring large shape changes [1]. To overcome such a difficulty, Eulerian-type methods employing a fictitious domain approach (see Fig. 1 (a)) with fixed grids have recently emerged as an alternative analysis tool for shape optimization.



(a) (b) (c) (d) (e)

Fig. 1. A fictitious domain and various Eulerian-type methods

Fig. 1 (c) shows a typical popular fixed-grid modeling with which an element stiffness is evaluated by the area fraction concept [2]. The concept is to evaluate the boundary element stiffness proportionally to the area fraction of the real domain ω within the boundary element. However, the boundary curve in this method needs to be approximated by zigzags that consist only of vertical and horizontal lines, so this area-fraction-based approach is not effective for curved boundaries. The only way to obtain accurate solutions near the boundary is to employ highly-dense grid distributions.
In order to obtain accurate solutions near the curved boundary without excessive grid densities, Jang *et al*. [3] have recently proposed a more direct approach: a curved boundary is approximated by piecewise oblique lines formed by the connections of the points between the curved boundary and the fixed grid lines. In this case, the piecewise oblique line, the approximated boundary curve, does not usually pass through the analysis nodes.
In this paper, we derive the analytic sensitivity analysis for the fixed-grid shape optimization based on the oblique boundary curve approximation [4]. Some results obtained by Hansen *et al*. [5] are used for the present analysis since the analysis grids not interacting with the boundary curves are stationary or fixed during the whole design process as in the method by Hansen *et al*. [5].

SENSITIVITY ANALYSIS

The total potential energy for a problem involving kinematic constraint is

$$\Phi = \int_{\omega} \left\{ \frac{1}{2}\mathbf{e}^T\mathbf{C}\mathbf{e} - \mathbf{f}^T\mathbf{u} \right\} d\omega + \int_{\Omega\backslash\omega} \varepsilon \frac{1}{2}\mathbf{e}^T\mathbf{C}\mathbf{e}\, d\omega \quad \text{with} \quad \mathbf{u} = \mathbf{0} \;\; \text{on} \;\; \Gamma_{\omega}^{g} \tag{1}$$

where $\mathbf{e}$ is the strain vector, $\mathbf{f}$ is the force vector, and $\mathbf{u}$ is the displacement vector. In the above expression, ε denotes a small positive value.
The design derivative of the first variation of equation (1) is given as

$$\delta\bar{\Phi} = \left[\int_{\omega} \delta\mathbf{e}^T \mathbf{C}\bar{\mathbf{e}}\, d\omega + \int_{\Omega\backslash\omega} \varepsilon\delta\mathbf{e}^T\mathbf{C}\bar{\mathbf{e}}\, d\omega \right] + \left[\int_{\omega} \left\{\delta\bar{\mathbf{e}}^T\mathbf{Ce} - \mathbf{f}^T\delta\bar{\mathbf{u}}\right\} d\omega + \int_{\Omega\backslash\omega} \varepsilon\delta\bar{\mathbf{e}}^T \mathbf{Ce}\, d\omega \right]$$
$$+ \int_{{}^D\Gamma_\omega} \left\{\delta\mathbf{e}^T\mathbf{Ce} - \mathbf{f}^T\delta\mathbf{u}\right\}\mathbf{V}^T\mathbf{n}\, d\Gamma - \int_{{}^D\Gamma_\omega}\left\{\varepsilon\delta\mathbf{e}^T\mathbf{Ce}\right\}\mathbf{V}^T\mathbf{n}\, d\Gamma = 0 \qquad (2)$$

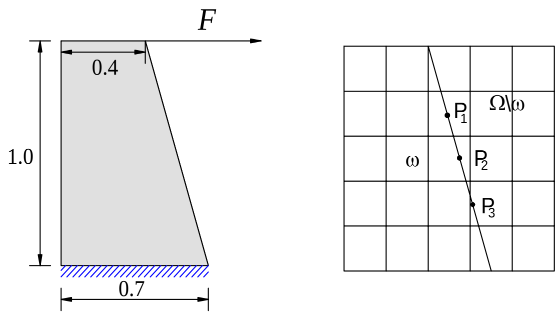
where $\mathbf{V}$ and $\mathbf{n}$ are the design velocity and the outward normal vector of the design boundary ${}^D\Gamma_\omega$, respectively. Unless the kinematic constraint is imposed on the design boundary, $\delta\mathbf{u}$ and $\delta\bar{\mathbf{u}}$ are in the same vector space, so that the terms in the second bracket in equation (2) vanish and the stiffness matrix for the sensitivity equation is exactly the same as that of the reference problem.
The last two terms in equation (2) form the force vector of the sensitivity equation after numerical formulation:

$$\underline{\tilde{\mathbf{f}}}^e = -\int_{-1}^{1} \left\{\mathbf{B}^T\mathbf{CB}\underline{\mathbf{U}}^e - \underline{\mathbf{N}}^T\mathbf{f}\right\}\mathbf{V}^T\mathbf{n}\,\tilde{J} d\zeta + \int_{-1}^{1} \varepsilon\left\{\mathbf{B}^T\mathbf{CB}\underline{\mathbf{U}}^e\right\}\mathbf{V}^T\mathbf{n}\,\tilde{J} d\zeta \qquad (3)$$

where $\underline{\mathbf{U}}^e$ is the nodal displacement vector of the element and $\underline{\mathbf{N}}$ is the matrix with bilinear shape functions.

NUMERICAL EXAMPLE

A structure in Fig. 2 (a) is considered as a numerical example. The bottom edge of the structure is fixed and a horizontal force $F = 1.0 \times 10^5$ is imposed on the upper right corner. The material has Young's modulus $E = 2.0 \times 10^8$ and Poisson's ratio $\nu = 0.3$. The fixed mesh in Fig. 2 (b) is used for this problem. The design sensitivities of the Von Mises stresses at the points of interest are listed in Table 1. The present analytic sensitivities agree well with the numerical results by the finite difference method (FDM).



(a) (b)

Fig. 2. A cantilever example and its fixed grids on the fictitious domain

Table 1. Sensitivities of the Von Mises stresses and their comparison with the finite difference results.

	Present	FDM
P_1	4.3407e6	4.3405e6
P_2	-4.3989e6	-4.3987e6
P_3	-9.8340e6	-9.8337e6

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