CREEP RUPTURE AND FIBER BREAKS ACCUMULATION IN UNIDIRECTIONAL COMPOSITE

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Summary *In this work we consider a model, based on Markov's-type stochastic kinetic equations for concentrations growth of adjacent fibers breaks in material under creep rupture conditions. Different from previous investigations, precise variety of all possible clusters geometrical configurations were discussed and form-dependent cluster distributions were obtained. Simultaneously, Monte Carlo simulations were performed.*

MATERIAL MODELING

Interest in precise damage accumulation modeling, for fiber composites subjected to long-term (creep-rupture) loading conditions, grown up last years with a new area of composite materials application. High corrosion resistance and competitive market price are making composites (especially polymer matrix composites) very promising in civil engineering application. We consider unidirectional composite material with parallel continuous fibers embedded in matrix. C*hain of bundles* material model is accepted.

Kinetic equations. Modeling of damage accumulation in composite material, under an arbitrary external tensile load is based on the assumption, that failure is a complex stochastic process, starting with scattered, isolated fiber brakes (at fiber's internal flow or structure heterogeneity) overstress redistribution on adjacent to broken fibers, failure of overstressed neighbors, forming breaks clusters and the breaks clusters growth and coalescence in the range of each bundle, orthogonally to the fiber direction. This process starting relatively slowly will transforms to catastrophic ultimate cluster growth, when overstress distributed on closest unbroken neighbors will immediately initiate one of the next fiber break. We can introduce two random variables $I(\sigma,t)$ and $J(\sigma,t)$ with joint probability function: $P\{I(\sigma,t)=i, J(\sigma,t)=j\} = H_i^j(\sigma,t)$, $\sigma \geq 0, t \geq 0$, to find cluster which was born and after that was stayed conservative till time moment *t*, as the cluster consisting of i adjacent fiber breaks and having a form number *j*. $W(\sigma,t)$ is a probability of fiber failure. For example (see Fig.1.), one and two fiber breaks have only one geometrical configuration, three adjacent breaks, in material with hexagonal fiber array, may form three different geometrical configurations (two different configurations for square array).

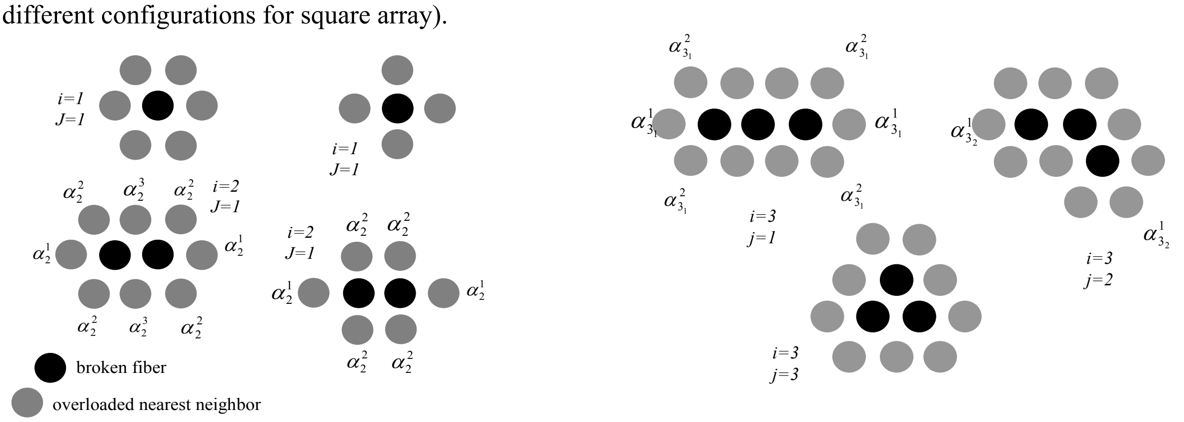

Figure1. Configurations of $i = 1,2,3$ adjacent broken fibers. $\alpha_{i_k}^j$ - overloaded nearest fiber.

For hexagonal packing, if the geometrical configuration of three adjacent breaks is forming line, we are designating it like cluster number one (form number $j=1$). Cluster with geometrical configuration forming curved line will have number two ($j=2$) and cluster with compact configuration - number three ($j=3$). For square packing we have $j=1,2$. Chance for double break cluster to grow and form tree break cluster with form index equal to number one, is the chance to fail one of two overstressed fibers belonging to the set of closest neighbors around two broken fibers (designated as α_2^1 in Fig.1). Here we must to note that overstress distributed on adjacent fibers α_2^k, $k=1,2,3$ coming from two breaks is different for fibers around broken and depends on k. Two break cluster growth to three breaks cluster's set with different geometry, will happen by fiber element failure under different overstresses. Probability to obtain cluster, consists of r adjacent fiber breaks and have form number j, is:

$$H_r^j(\sigma,t) = \sum_{l:(r-1)_l+\alpha^k_{(r-1)_l}\to r_j} \int_0^t \frac{dH^l_{r-1}(\sigma,t^*)}{dt^*} m^k_{(r-1)_l} W\left(k^k_{(r-1)_l}\sigma, t-t^*\right)\left(1-W\left(k^k_{(r-1)_l}\sigma, t-t^*\right)\right)^{m^k_{(r-1)_l}-1} dt^* \quad . \tag{1}$$

These are kinetic equations, in recurrent form, for stable states integral probabilities. Equations are explaining the stochastic process of damage accumulation in loaded unidirectional composite material. Applying constant tensile stress $\sigma = const \neq \sigma(t)$ to composite material, according to statistics of extremes we appropriate Weibull type integral probability function for an element failure. Fiber breakage leads to local stress redistribution in vicinity of failure place. Overstresses on adjacent to broken fibers was calculated using Hedgepeth-Van-Dyke shear lag model and discrete Fourier transform [1].

Lower tail analysis. Using non-effective length δ_0 like the governing parameter for material partition we obtain a large number of fiber elements in any real mechanically loaded composite volume. That's mean we are interesting in a small clusters probabilities or in a lower tail for probabilities distributions. Looking on obtained formulae (1) we can conclude that all clusters probabilities $H_r^j(\sigma,t)$ can be represented multiplying function dependant only on stress and geometry by function dependent on time $H_r^j(\sigma,t) = H_r^j(\sigma)\cdot H_r(t)$. Time dependant integrals $H_r(t)$ can be transformed to incomplete Beta – functions and be written in a form convenient for easy numerical analysis.

NUMERICAL RESULTS

A numerical example of damage accumulation was performed for Kevlar 49/Epoxy. When we calculate the probabilities for clusters with different configurations, some of these configurations are more probable then others. These configurations called dominant. Calculations shown, the probabilistic modeling is sensitive to overstress coefficient (the load concentration factor). Using the dominant cluster approach (a technique widely used by different authors), instead of precise calculations for all configurations, results may be grossly inaccurate. An isolated single broken fiber has six overloaded neighbors with overstress on them taking into account by the same overloading coefficient. Surrounding two broken fibers, we have eight overloaded neighbors: two of them loaded with overstress k_2^1, four have coefficient k_2^2, and another two k_2^3. For three adjacent breaks, there are eleven neighboring overloaded fibers, each having an overstress factor and multiplicity. For four adjacent breaks, we have 46 differently overloaded fibers (for small clusters $r = 1 \div 7$ number of their geometrical configurations was calculated in [2]). It is apparent that the variety of clusters grows rapidly as the number of adjacent broken fibers increases (10 break clusters have a little more than 30000 different configurations, and 11 break clusters have more than 140000 configurations; every particular geometrical configuration has differently overloaded fibers around it), increasing the number of mathematical calculations. We performed precise analysis until the cluster consists of 11 broken fibers. Further, analytical approximation was used. Comprehensive damage accumulation picture was obtained. Parametric analysis was made. Bundle stiffness degradation was appreciated.

Model validation. In order to evaluate model capability, Monte Carlo simulations for fiber break accumulation in composites under creep-rupture loading were conducted. Both methods predict the same small, different configuration, cluster accumulation. In the vicinity of the unstable growth, the situation changes. Clusters with slower growing speed cannot realize all their possible configurations. Predicted in model ultimate cluster appearance lifetime usually would be shorter then in reality (and obtained in Monte-Carlo simulations), if we will look on summarized probabilities curves. That mean, on the stage of formation of ultimate cluster, for its appearance time evaluation, we must to go from the summarized probabilities curves to one dominant cluster growth (this cluster size we designate as ultimate).

CONCLUSIONS

Fiber break accumulation in unidirectional composite material under creep-rupture loading conditions was investigated using kinetic theory and Monte-Carlo simulations. Detailed damage accumulation picture of all varieties of geometrical configurations for fiber break clusters were obtained and analyzed. Material stiffness degradation and volume lifetime prediction was made and compared with experiment. Parametric analysis was made.

References

[1] Bandorawalla, T. Micromechanics-based strength and lifetime prediction of polymer composites, Ph.D. dissertation, Virginia Polytechnic Institute and State University, (2002), http://scholar.lib.vt.edu/theses/available/etd-03172002-151608/

[2] Smith, R. L., Phoenix, S.L., Greenfield, M.R., Henstenburg, R.B. and Pitt, R.E. Lower-tail approximation for the probability of failure of three-dimensional fibrous composites with hexagonal geometry. Proc. Roy. Soc. London., A388 (1983) 353- 391.