DYNAMIC INSTABILITY OF A HIGH-SPEED FLEXIBLE SHAFT WITH A MASSIVE DISC AND FOLLOWER LOAD

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Extended Summary

The paper is motivated by design and maintenance problems of rotating machinery like turbines, compressors, fans, etc. A dynamic analysis is presented concerned with transverse flutter instability and vibration of a flexible rotating shaft supported like a cantilever beam, carrying a thin massive end-disc and subject to concentrated follower load exerted on the disc. The follower load is a consequence of fluid-structure interaction in real turbomachinery. A working medium flow can produce either compressive or a tensile loads, therefore unlike in Beck's problem, the follower load is considered both positive and negative.

Internal dissipation in the rotating shaft and follower load can cause dynamic flutter-type instability and bifurcation of the considered system. Both instability mechanisms are different in physical nature but mathematically modelled they interact to produce complicated stability domains in the space of rotation speed and follower load. [1].

The main question addressed in the present paper is the gyroscopic effect in the stability and flutter problem of the considered discrete-continuous system. It has been already shown that the gyroscopic effect can considerably change the scenario of bifurcation even in slender shafts. [2]. In case of massive disc mounted at the free end of a shaft the gyroscopic effect can change bifurcation dramatically.

The shaft carrying disc can be a composite structure. Laminated shafts are torsionally stiff and strong being simultaneously much lighter than similar structures made of conventional materials such as steel. Such a system can easily meet requirements concerning torsional strength and stiffness but are more flexible to bending and show high level of internal friction. In consequence, high-speed laminated shafts can be especially endangered by dynamic instability and large-amplitude self-excited whirling.

In the considered discrete-continuous model coupling between cantilever beam and massive disc is realised through boundary conditions.

The shaft is assumed to be slender, transversely unloaded, of circular cross-section, rotating with a constant angular speed. Transverse shear is neglected but rotational inertia is included. External damping is assumed as linear. Internal damping in a generally anisotropic shaft material corresponds to the Kelvin-Voigt model when average using Kirchhoff's deformation hypothesis.

Transverse shaft vibration of the system can be described by the partial equations [2]

$$\rho A\left(\frac{\partial^2 u}{\partial t^2} + h^* \frac{\partial u}{\partial t}\right) = \frac{\partial}{\partial x}\left[T_Y\left(\sqrt{1 + \left(\frac{\partial u}{\partial x}\right)^2}\right)^{-1}\right] - P\frac{\partial^2 u}{\partial x^2}$$
$$\rho A\left(\frac{\partial^2 v}{\partial t^2} + h^* \frac{\partial v}{\partial t}\right) = \frac{\partial}{\partial x}\left[T_Z\left(\sqrt{1 + \left(\frac{\partial v}{\partial x}\right)^2}\right)^{-1}\right] - P\frac{\partial^2 v}{\partial x^2} \tag{1}$$

where, T_Y, T_Z denote shear resultants determined as follows:

$$T_Y = -\frac{\partial M_Z}{\partial x} + \rho I_Z \frac{\partial^3 u}{\partial x \partial t^2} + \rho I_X \Omega \frac{\partial^2 v}{\partial x \partial t};$$
$$T_Z = -\frac{\partial M_Y}{\partial x} + \rho I_Y \frac{\partial^3 v}{\partial x \partial t^2} - \rho I_X \Omega \frac{\partial^2 u}{\partial x \partial t};$$

where in turn u, v denote transverse displacements in perpendicular directions, ρA is shaft mass per unit length, h^* - viscous external damping coeffcient, Ω - angular speed of the rotor, M_Y, M_Z - bending moments: $M_Y = \int \sigma Z dA$, $M_Z = \int \sigma Y dA$, while σ denotes normal stress in the shaft cross-section.

We shall assume stress-strain relation in the global form of the Kelvin-Voigt model, i.e.

$$\sigma = E_0(1 + \beta_0 \frac{\partial}{\partial t})\varepsilon \tag{2}$$

Discretisation of partial equations is performed using two cantilever beam functions satisfying mass-dependent boundary conditions. Two boundary eigen-values problem has been solved numerically by means of FEM.

Liapunov criterion of stability applied to discretised system leads to the stability region in the space of active parameters.

$$\Psi = \{(\mu, \omega) : Re\, r_i \le 0 \ (i = 1,...,8)\} \tag{3}$$

where r_i denotes eigenvalues of the linearised system matrix, ω is demensionless rotation speed and μ demensionless follower load.

The non-linear flutter analysis of the system is made using Iooss's approach to determine bifurcation diagrams in terms of rotation speed and flutter amplitude under fixed but changeable follower load.

It is shown that the gyroscopic effect can influence the stability domain of the rotor, especially in highly loaded, weakly internally dissipative systems.

In some regions of the system parameter space the gyroscopic effect can considerable change the near-critical rotation speed can be increased, but simultaneously bifurcation can be changed from super- to subcritical, producing unstable limit cycles in the stable domain of equilitium, as shown in Fig. 1.

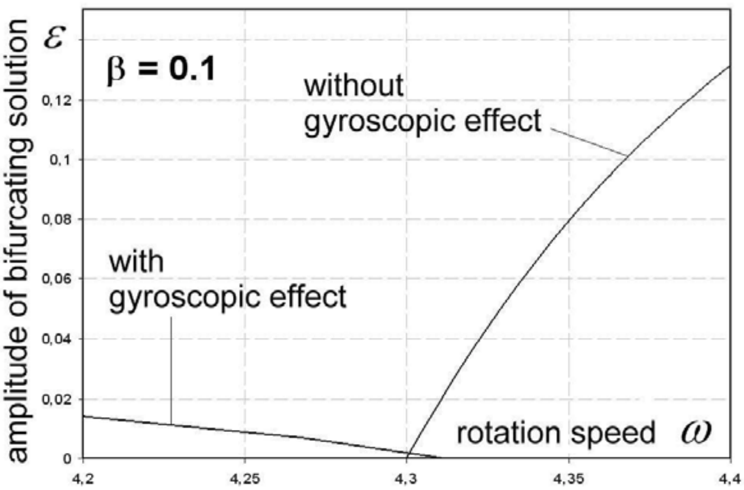

Fig. 1. Bifurcation diagram of the system with gyroscopic effect neglected and included

References:

[1] Kurnik, W., 1994, Stability and Bifurcation Analysis of a Non-linear Transversally Loaded Rotating Shaft, *Nonlinear Dynamics,* 5, No. 1, 39-52.

[2] Kurnik, W., Weronko, J., 2003, Optimisation of a flexible cantilever rotor with intermediate support, *5th EUROMECH Solid Mechanics Conference, Thessaloniki GREECE,* Book Of Abstract, p. 155.