ON STABILITY OF SYSTEMS SUBJECT TO GENERALIZED FOLLOWER FORCE

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Summary. Stability and dynamics of columns and whirling shafts subject to a generalized follower force is considered in the paper. The generalized follower force is defined as one, that moves with the body on which it acts, and that always preserves the same attitude to the body as it moves. The both linear and nonlinear analysis are included. The first one relates to the influence of system parameters on the system response whereas the second one is focus on the near critical behaviour of the system under both tension and compression loads including analysis of the corresponding limit cycle.

DESCRIPTION OF THE MODEL

The paper relates to stability and dynamics of slender self-excited structures. The considered structures, columns and whirling shafts, are subject to a generalized follower force that is defined as ones that moves with the body on which it acts, and that always preserves the same attitude to the body as it moves. The model of loading is defined in Fig.1 on example of column structure for which the force inclination χ and eccentricity e, depend on the actual inclination α and displacements f of the structure at the point of the force application.

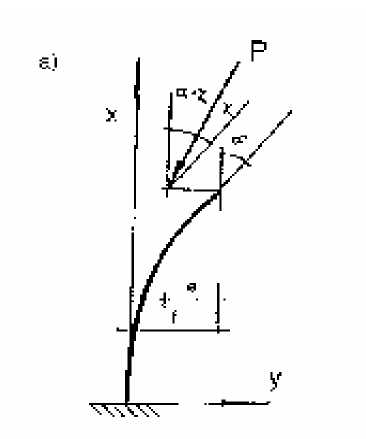

Fig.1. Model of generalized force

$$e = \rho l\alpha + \vartheta f, \qquad \chi = \mu\alpha + \gamma f / l, \tag{1}$$

where $\rho, \vartheta, \mu, \gamma$ are the non-dimensional parameters and l is the length of the characteristic dimension of the element to which the force is applied. An interaction between two surfaces in relative motion like the contact force of the rail and the wheel; a force attached to the structure through any connector that moves due to structural deformation like the bearing of a bridge span; the support force of rotating shafts subject to vapour pressure, like in Laval rotor or loading of the fluid conveying pipes are examples of the considered forces. The tangential concentrated loading is the case of force defined by Eq.(1). Herein, the models of the elastic and viscoelastic columns and whirling shafts are treated in details. The analysed boundary value problem is formulated as following

$$\mu\ddot{w}(x,t) + F_x[P, w(x,t)] + \beta\dot{w}(x,t) = 0, \tag{2}$$

plus homogeneus boundary conditions

where $w=w(x,t)$ is the transverse displacement, μ is the mass density, P is the compressing force (control parameter), β is the damping coefficient and $\ddot{w}$ is the acceleration, $\dot{w}$ is the velocity of the variation of w. The precise equation written for column or whirling shaft are in the class of Eq.(2). Notice that the conservation of the system energy depends on the differential operator F_x, that relates to the internal and the external forces. The selfadjointness of the operator F_x in with respect to the boundary conditions ensures the conservation of the system energy. The condition of

$$\int_0^l w(x,t)F_x[P, v(x,t)]dx = \int_0^l v(x,t)F_x[P, w(x,t)]dx, \tag{3}$$

where w and v are two sufficiently smooth function, which satisfy the boundary conditions is fulfill for conservative system. Also, the condition (3) ensures that the system becomes unstable by divergence.

The next feature of the system, namely existence of the potential, results from the condition of the rotation equal to zero, rot$\mathbf{v}=0$, from which one gets

$$\frac{\partial M}{\partial f} = \frac{\partial H}{\partial \alpha}, \tag{4}$$

where $M = Pe$ and $M = P(\alpha - \chi)$.

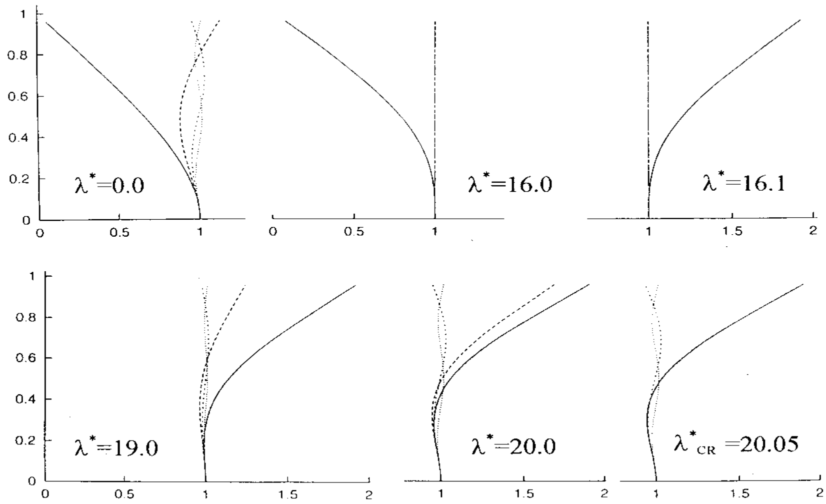

Fig.2. Eigenmodes of column subject to tangential loading, for different values of loading λ^*

RESULTS OF ANALYSIS

The study is divided into two parts, relating to linear and nonlinear analysis respectively. The influence of system parameters on the response of the system is consider for the linear model. It occurs that the shape of the eigenmodes depends of the value of loading, namely the first mode can alter into the second one for high enough value of force. In that case the second eigenmode occurs at the critical point with the fundamental frequency. The phenomenon is explained Fig.2, where λ^* is the nondimensional force. Notice that at the point of eigenmode change, for $\lambda^*= 16$, the displacements which relates to higher eigenmodes diminishes whereas the sign of displacements of the first eigenmode switches. On force-frequency plane the point of eigenmode change is observed as the point of intersection of eigencurves that relates to different values of system parameters creating so called fixed point, for which the frequency of natural vibration occurs independent on a variety of the system parameters [1].
The described phenomenon is observed in different types of structures: columns, slender whirling shafts etc., subject to follower type of loading
The variation of the load inclination χ and eccentricity e, described by expression (1) may follow the special design of the supporting condition or can be forced by an additional source of energy, for conservative and nonconservative systems repectively. However the phenomenon of eigenmode change is independent on the system conservativeness or nonconservativeness and come across both types of systems. Notice, that the high value of the critical force observed for structures subject to follower type of loading, that can be 8-10 times higher than for Euler column, can be explained by the higher eigenmode which occur at critical state. Due to the Euler formula the shorter buckle length relates to higher value of critical force. Moreover, in nonconservative systems the flutter point occurs at the double eigenvalue for which the shape of respective eigenforms is exactly the same. Some examples from literature shows that the problem is often not properly touched by researches.

The linear analysis is completed by nonlinear one where an interesting effect is related to the double Hopf bifurcation which consists in simultaneous loss of stability by two eigevalues. On example of the model of whirling shafts supported by fluid film bearings it is shown as the two critical eigevalues enter the instability domain at generally different rotation speeds. It corresponds to a sequence of Hopf bifurcation initiating self-exited vibration induced either by the shaft or by the bearing [1]. The consideration is completed by an additional feature relating to the considered class of structures. On example of Leipholz column, loaded by distributed compressive follower forces, it is shown that flutter can occur either as a supercritical bifurcation into a stable limit cycle when an model of slight internal damping is considered or a subcritical bifurcation into an orbitrary unstable limit cycle existing in the stable region of equilibrium in the case of intensive internal damping [2].

References

[1] W.Kurnik, Stability of a Viscoelastic Shaft in Fluid-Film Bearings, Machine Dynamics Problem, 1999.v.23,no.4, 39-52

[2] W.Kurnik, M.Pekalak, Stability And Bifurcation Analysis Of The Non-Linear Damped Leipholz Column, Journal of Sound and Vibration, 1992, 152(2,285-294)

[3] W.Kurnik, Divergent and oscillatory bifurcations, WNT, Warszawa, 1997

[4] R.Bogacz, Sz.Imielowski, On Eigenmodes of Columns Subject to Generalized Follower Force, 2003, AIAA Journal /in print/