MULTIDIMENSIONAL MODEL OF COMBINED DRY FRICTION

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Summary. A new multidimensional model of combined dry friction is presented. This model is generalization of primary one-dimensional Coulomb model to the case when relative motion of rubbing solids is combination of sliding and spinning for arbitrary form of area of contact. The method of straight constricting of quadratic Pade approximation of components of friction force and momentum is developed for using of the proposed model in the task of solids dynamics

INTRODUCTION

Contensou in case of not point contact of the moving bodies had established the correlation between the friction of slide and the friction of spin [1]. Using the Hertz theory of contact, he had got the numeric function of the dry friction force from ration of the slide and rotation velocities, supposing that the both contiguous surfaces are local spheres. But Contensou had restricted the calculation by the friction force only and not considered the momentum of the friction forces.

Zhuravlev had significantly improved the Contensou theory [2]. He had got exact analytical functions of the dry friction force and momentum from ration of the slide and rotation velocities. To use these relations in the tasks of dynamics of rigid bodies, Zhuravlev had built the partial-linear Pade approximation of the dry friction force and momentum.

Kireenkov had applied Contensou - Zhuravlev theory to solve the task about motion with rotating of the flat disk on the plane [3]. He had got exact functions and Pade approximations of the dry friction force and momentum, supposed that area of contact is of the circle form, while distribution of the normal contact stress in the area of contact has form. Then Kireenkov has developed a direct method for constructing of partial-linear Pade approximation of friction force and momentum for area of contact having circle form as a function of ration of the slide and rotation velocities [4].

On the base of describing above investigations, Zhuravlev has built a two-dimensional model of combined dry friction [5]. He has developed a direct method for constructing partial-linear Pade approximations of friction force and momentum in term of velocities of sliding V and rotation ω for the case of the circle area of contact and arbitrary axial-symmetric distributions of contact stresses. Using partial-linear Pade approximations for force and momentum, Zhuravlev has constructed a “STANDSTIL” zone which is very convenient for qualitative describing the dynamics of solids when either friction force or momentum are tended to zero.

The suggested below multidimensional model of combined dry friction is logical generalization of two-dimensional Zhuravlev model on the case of arbitrary form of area of contact and arbitrary distributions of contact stresses.

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Under the words “model of dry friction” we understand the interrelations between forces, momentums and velocities. The physical nature of phenomena within the area of contact is not interested for us. It is supposed that the form of area of contact and distributions of contact stresses are known.

For the model constructing we used following basic suppositions:

- Arbitrary form of area of contact
- Arbitrary distributions of contact stresses
- Relative motion of friction solids is combination of sliding and spinning:

In suggestion of validity of the Coulomb low in local form for differential of the tangent friction force at any elementary square within the area of contact we have the following relations between force, momentum and velocities:

$$F_x(V,\omega) = -f_0 \iint \frac{(V-\omega\, y)\,\sigma(x,y)}{\sqrt{\omega^2\,(x^2+y^2) - 2\,\omega\, V y + V^2}}\, dx\, dy$$

$$F_y(V,\omega) = -f_0 \iint \frac{\omega\, x\,\sigma(x,y)}{\sqrt{\omega^2\,(x^2+y^2) - 2\,\omega\, V y + V^2}}\, dx\, dy$$

$$\mathrm{M}(V,\omega) = -f_0 \iint \frac{(\omega\,(x^2+y^2) - V y)\,\sigma(x,y)}{\sqrt{\omega^2\,(x^2+y^2) - 2\,\omega\, V y + V^2}}\, dx\, dy$$

where $F_x(V, \omega)$ - component of the friction force in the direction of the vector of velocity of sliding V, $F_y(V, \omega)$ - component of friction force in perpendicular direction to the vector of velocity of sliding V, $M(V, \omega)$ - momentum of dry friction, $\sigma(x, y)$ - distribution of contact stresses, f_0 - coefficient of friction.

These expressions as function of V and ω are invariant with respect to similarity group. Consequently, any approximations for the friction force and momentum have to be invariant with respect to similarity group. To use these expression in the tasks of solids dynamics quadratic Pade approximations have constructed. Utilizing of quadratic Pade approximations is substantiated by the requirement simultaneously describing of the behavior of function at zero and infinity and insufficiency of partial-linear Pade approximations to approximate the component of force of friction in the perpendicular direction to the vector of velocity of sliding V. To calculate corresponded Pade approximations we choose the origin of coordinate systems from the conditions of conversion to zero the momentum of friction forces simultaneously with the velocity of spin ω: $M(V, 0) = 0$. This point is fixed in the area of contact and may be interpreted as the center of gravity of the contact spot in the supposition that the distribution of normal stresses is playing the role of density.

$$\iint_D x\sigma(x, y)dxdy = 0, \quad \iint_D y\sigma(x, y)dxdy = 0$$

Quadratic Pade approximations for force and momentum of dry friction

Quadratic Pade approximation, conserved the behaviour of the force and momentum and it's the first, the second and the third derivations in zero and infinity have form

$$F_{xp} = \frac{F_0(v^2 + a_1 v\omega)}{v^2 + a_1 v\omega + a_2\omega^2}, \quad F_{yp} = \frac{b_1\omega^2}{b_2 v^2 + b_3 v\omega + \omega^2}, \quad M = \frac{M_0(\omega^2 + m_1 v\omega)}{m_2 v^2 + m_1 v\omega + \omega^2}$$

where corresponded coefficients have been calculated.

In the case of axially symmetric areas of contact (circle, ellipse, rectangular) the force component F_y is zero. Consequently, the quadratic Pade approximations are significantly simplified, and have form

$$F_{xp} = \frac{F_0(v^2 + a_1 v\omega)}{v^2 + a_1 v\omega + a_2\omega^2}, \quad M = \frac{M_0(\omega^2 + m_1 v\omega)}{m_2 v^2 + m_1 v\omega + \omega^2}$$

where coefficients are depend on the turn angle of the rotation solid respectively choosing system of coordinate.

Formulas defined the corresponded quadratic Pade approximations may be considered as the rheological model of combined dry friction.

CONCLUSIONS

- Quadratic Pade approximations are required to describe the appearance of component of dry friction force in perpendicular directions to vector velocity of sliding.
- Only quadratic terms contained in Quadratic Pade approximations.
- It is not needed to calculate the integrals, defining the coefficients of Pade approximation. The corresponded coefficients may be defined from experiments.

References

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