# PARALLEL MULTILEVEL SOLUTION OF LARGE SCALE NONLINEAR SHELL PROBLEMS

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<u>Summary</u> The conditioning problem of thin–walled structures like plates and shells is still a major obstacle for iterative solvers. In the present contribution two procedures to remedy this difficulty are discussed. The first one is a mechanically oriented preconditioner, used for a three–dimensional shell formulation called Scaled Director Conditioning. The second procedure utilizes a parallel multilevel approach which is based on an aggregation concept.

#### INTRODUCTION

The analysis of large–scale nonlinear shell problems asks for parallel simulation approaches. One crucial part of efficient and well scalable parallel FE–simulations is the solver for the system of equations. Due to the inherent suitability for parallelization one is very much directed towards preconditioned iterative solvers. However thin–walled structures like plates and shells suffer from severe ill–conditioning due to the extreme stiffness differences attributed to membrane, shear and bending action. This is even worse when thickness change effects are included in a sense of a three–dimensional shell formulation. This of course becomes a severe handicap when large scale nonlinear problems ought to be handled by iterative solvers. A preconditioner for this challenging class of problems is presented combining two approaches in a parallel framework.

#### SCALED DIRECTOR CONDITIONING

The first approach is a mechanically motivated improvement called Scaled Director Conditioning (SDC) [4],[5] and is able to remove the extra – ill conditioning that appears with three dimensional shell formulations [1], compared to formulations that neglect thickness changes of the shell. It is based on the scaling of the director so that the conditioning reaches that of classical Reissner–Mindlin type of formulations. It is introduced at the element level, acts as an almost no–cost pre–preconditioner and harmonizes well with the second approach.

## AGGREGATION MULTIGRID CONCEPT

The second approach which essentially remedies the remaining ill–conditioning is a parallel multilevel approach. A hierarchy of coarse grids is generated in a semi–algebraic sense using an aggregation multigrid concept [2],[3]. On each grid level mutually disconnected patches (aggregates) are constructed from the system of equations and from some geometric information taken from the next finer grid. No coarse grid triangulations have to be supplied. The intergrid transfer operators  $\mathbf{P}_{(j)}^{(j+1)}$  (j=1 to n–1 levels) are built such, that the rigid body modes of each aggregate are represented exactly. Except for the coarsest level, each level supplies smoothers based on an overlapping domain decomposition with inexact solves on subdomains. On the coarsest grid, the correction is obtained via a direct exact solve.

### **EXAMPLES**

Both preconditioning approaches are combined and the performance of iterative solvers is demonstrated via some numerical examples. Figure 1 describes the bending of a tube and the iteration performance during the course of solver calls for a one and a four level preconditioning.

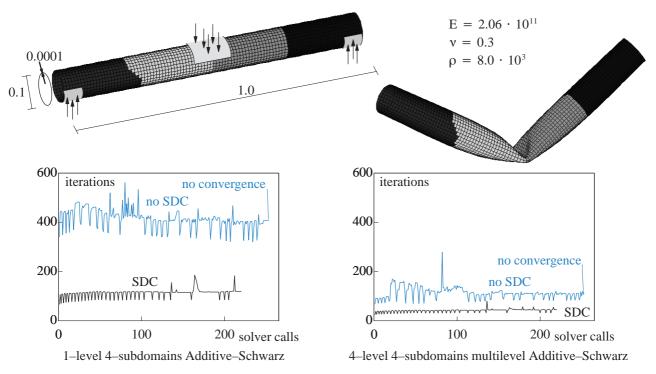


Fig. 1: Tube Bending

For the tube intersection the convergence and number of iterations for different numbers of levels within the aggregation multigrid are shown in figure 2.

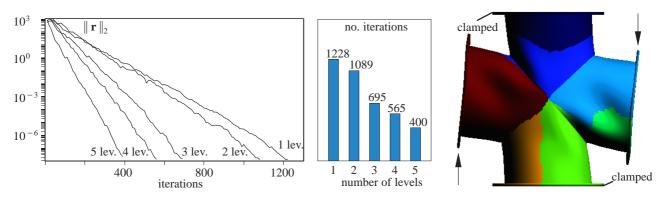


Fig. 2: steel tube intersection, 131.388 dofs, AMG-preconditioned CG, (exact solve on coarsest grid only with 5-level preconditioner)

In addition implementation issues concerning performance and parallelity of the algorithms for distributed memory machines will be discussed.

# References

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