APPLICATION OF THE RETURN MAPPING ALGORITHM TO PERZYNA VISCOPLASTICITY FOR PLANE STRESS

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Summary In the paper the elastic-viscoplastic constitutive relations of the Perzyna-type are investigated for plane stress problems. A fully implicit integration algorithm is adopted and the relevant expression for the consistent tangent operator for the von Mises yield criterion and flow functions of arbitrary type is derived. It is shown how the elasto-plastic rate equations of standard plasticity can be generalized to overstress-type models of viscoplasticity, where the stress point can be located outside the loading surface. Numerical example is given to reveal the differences and the similarities between the plastic and the viscoplastic overstress models.

PROBLEM FORMULATION

This paper presents a version of the return-mapping algorithm for plane stress problems. Three-dimensional radial return algorithm can be easily modified for the plane strain problem, but not for problems with additional constraints on stresses. Two such constraints are of interest for plates and shells: (a) the zero normal stress condition, $\sigma_{33} = 0$, and (b) the plane stress condition, $\sigma_{\alpha 3} = \sigma_{33} = 0$. With the latter constraints the radial return algorithm and an explicit expression for viscoplastic multiplier cannot be used (cf. Simo and Taylor [2], Alfano at al.[5]).
In literature, various viscoplastic material models have been proposed for the analysis of time-dependent deformations of materials. A widely-used viscoplastic formulation is the Perzyna model [1]. The main feature of this model is that the rate-independent yield function used for describing the viscoplastic strain can be larger than zero, the effect known as the 'overstress'. The characteristics of the Perzyna model as well as the numerical discretization have been addressed by various authors (e.g. Simo and Hughes[3], Ju [6], Klee and Paulun [7]).

PERZYNA VISCOPLASTICITY

The main idea of the viscoplastic flow mechanism is to accomplish in one model the description of behaviour of material valid for the entire range of strain rate changes. To achieve this aim the empirical overstress function Φ has been introduced and the strain rate is postulated in the form as follows (cf. Perzyna [1])

$$\dot{\epsilon}_{ij}^{vp} = \gamma \ \langle \ \Phi(\frac{\sigma_{eq}}{\sigma_Y(\epsilon_{eq}^{vp})} - 1) \ \rangle \ \frac{\sigma_{ij}^{'}}{\sigma_{eq}} \quad , \tag{1}$$

where σ_{eq} is effective stress, ϵ_{eq}^{vp} is effective strain, σ_Y is a yield stress and the overstress function is defined as

$$\Phi(\cdot) \ = \ (\frac{\sigma_{eq}}{\sigma_Y(\epsilon_{eq}^{vp})} - 1)^m \quad . \tag{2}$$

In equation(1) and (2) the material constants are $\gamma \in [0, +\infty]$ and $m \geq 1$. The symbol $\langle \ \rangle$ means $\langle \Phi \rangle = F$ for F>0 and $\langle \Phi \rangle = 0$ for $F \leq 0$.

NUMERICAL EXAMPLE

We analyze a thin rectangular strip with a circular hole in its axial direction, subjected to increasing extension in a direction perpendicular to the axis of the strip and parallel to the long side of the rectangular section. For symmetry reasons the analysis is performed for one quarter of the section with appropriate boundary conditions. The adopted mesh consists of 230 nodes and 198 elements. The material used was an aluminium $57S$ with elastic modulus $E = 7000$ MPa, Poisson's ratio $\nu = 0.3$, a yield stress $\sigma_y^0 = 243$ MPa. The strain hardening of the aluminium is idealized by a linear hardening function with the plastic tangent modulus $H = 220$ MPa, like in Theocaris and Marketos [4].
The ratio of the diameter d of the hole to the width w of the strip was taken equal to one half. The thickness, t, of the strip was taken equal to 1/56 of the width, w, which is much less than the radius of perforation d. In each analysis, the velocity $\dot{U}$, was fixed. Different velocities $\dot{U} = \Delta u / \Delta t$ are obtained by changing the time increment Δt and keeping Δu constant.
The return mapping algorithm for the Perzyna viscoplasticity model was implemented in ABAQUS via a user-defined material subroutine UMAT. We used a 4-node element with reduced Gauss integration $(CPS4R)$.

The obtained strain and stress distributions are compared with the numerical results of Klee and Paulun [7] and with the experimental data published by Theocaris and Marketos [4]. The following results of the numerical calculations are presented.

1. The isochromatic patterns in normal incidence for the ratio of the diameter of the hole to the width of the strip $d/w = 0.5$ (after Theocaris and Marketos[4] (Steps I-III-V-VI)) is compared with the distribution of the equivalent plastic stress obtained by our numerical method.

2. The evolution of elastic-plastic boundary for $d/w = 0.5$ is presented. The final plastic zones for the loads IV ($\bar{t} = 0.4\sigma_y^0$) and VI ($\bar{t} = 0.53\sigma_y^0$) obtained numerically by our method and Klee and Paulun [7] based on the FEM are compared with the experimental ones.

3. The distribution of stress σ_{22} in the minimum section of strip in dependence of time is given. The steady state can be compared with the measured values of Theocaris and Marketos [4].

A good agreement of the numerical results with the experimental ones is observed.

CONCLUSIONS

A non-linear solution procedure based on the backward-Euler operator for the rate dependent Perzyna-type models is presented. The plane stress return-mapping formulation for a general overstress viscoplasticity models is described, and it has following features:

- the stresses are updated in an efficient and reliable way;
- in one algorithm we can efficiently handle the viscoplastic and plastic plane stress cases;
- the derived consistent tangent operator gives the appropriate convergence rate in the Newton-Raphson solution procedure. For other viscoplastic models (e.g. Duvaut-Lions), results can be readily obtained by specializing the relevant flow and overstress function. In the absence of viscous effects, i.e. when $\gamma\Delta t \longrightarrow \infty$ the consistent plastic tangent operator for plane stress is recovered;

Numerical computations carried out for the typical benchmark problem and comparison with the experimental data of Theocaris and Marketos [4] confirmed the accuracy and robustness of the proposed algorithm.

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