MINDLIN CYLINDRICAL PANELS WITH TWIST AND DOUBLE CURVATURE

T. Sakiyama[a], C. W. Lim[b], X. X. Hu[b,c], H. Matsuda[a], C. Morita[a]
[a] *Nagasaki University,1-14 Bunkyomachi, Nagasaki 852-8521, Japan*
[b] *City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong*
[c] *Zhejiang University of Technology, Hangzhou, Zhejiang 310032, PRC*

Summary An open cylindrical panel, which has a twist around the lengthways direction and double curvature in the radial and lengthways directions, is a better shell model of some turbine blades. For the sake of high accuracy of a vibration analysis method, a precise relationship between strains and displacement components of the shell model is derived on the general shell theory and the first order shear deformation theory. By using the principle of virtual work and the Rayleigh-Ritz method, the governing equation for free vibration of the model is presented. The effects of parameters such as curvature and twist on vibration are studied.

INTRODUCTION

Several comprehensive surveys of the research work on vibration of blades have been done by Rao [1-3] and Leissa [4,5], respectively. It is known that three models such as beam, plate and shell were adopted for studying the dynamics of blades, among of them the beam was the most common model and is also used for the research of blades now. Although the plate and the shell models were more approximate to the geometric configurations of blades, and there was a little related work reported. Since 80s more complicated and precise shell models have been presented. For example, Leissa, Lee *et al.* used twisted plate, shallow cylindrical shell and doubly curved shallow shell for models of turbo machinery blades and studied their vibration performance by the Ritz method [6-9]. Introducing a *pb*-2 Ritz method, Lim *et al.* studied the problem using several models such as the trapezoidal plate, the shallow cylindrical shell and the shallow conical shell [10-13]. Based on an exact strain-displacement relationship on the general shell theory, Tsuiji *et al.* presented a pretwisted thin plate and thin cylindrical models [14,15], and Hu *et al.* presented a curved and twisted thin cylindrical model [16] for the problem. Recently, Sakiyama *et al.* studied further the vibration characteristics of the twisted cylindrical panel with non-uniform thickness and the parametric effects [17].

The purpose of this work is to extend the research of thin shell model and to introduce an accurate strain-displacement relationship of the model under considering the influence of shear deformation. By the use of the principle of virtual work the energy equation of the panel for vibration is given out, and then the governing equation is achieved by the Rayleigh-Ritz method with orthonormal polynomials as admissible displacement components. The vibration characteristics and the effects on it are investigated briefly.

THEORETICAL FORMULATION

A profile, geometric parameters and the coordinate system of a new blade model is shown in Figure 1 where the cylindrical panel twists around the curvilinear *x*-axis at an constant twist rate *k*, and has two curvature (1/*R* and 1/*r*). Based on the accurate relation between strains and displacement components for the 3D panel [16] and the first order shear deformation theory, an exact strain formulae for the Mindlin cylindrical panel are obtained as follows:

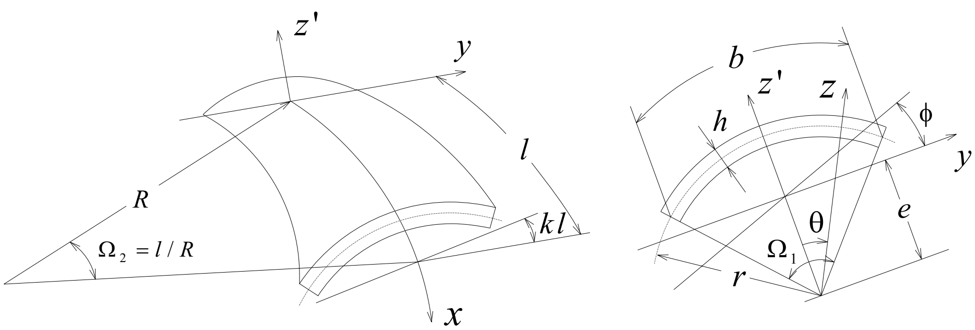

Figure 1. A schematic diagram of a Mindlin cylindrical panel

$$\left\{ \varepsilon_{\xi\xi} \quad \varepsilon_{\eta\eta} \quad \gamma_{\xi\eta} \quad \gamma_{\xi\zeta} \quad \gamma_{\eta\zeta} \right\}^T = \mathcal{Z}_1 \mathcal{G}^{(1)} \mathcal{R}_1 + \mathcal{Z}_2 \mathcal{G}^{(2)} \mathcal{R}_2, \tag{1}$$

$$\mathcal{R}_1^T = \left\{ \frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial y} \quad u \quad \frac{\partial v}{\partial x} \quad \frac{\partial v}{\partial y} \quad v \quad \frac{\partial w}{\partial x} \quad \frac{\partial w}{\partial y} \quad w \right\}, \qquad \mathcal{R}_2^T = \left\{ \frac{\partial \psi_x}{\partial x} \quad \frac{\partial \psi_x}{\partial y} \quad \psi_x \quad \frac{\partial \psi_y}{\partial x} \quad \frac{\partial \psi_y}{\partial y} \quad \psi_y \right\}, \tag{2}$$

where $\mathcal{G}^{(1)}$ and $\mathcal{G}^{(2)}$ are matrices relative to the geometric parameters, matrices $\mathcal{Z}^{(1)}$ and $\mathcal{Z}^{(2)}$ are related to the thickness of panel only, and *u, v, w* are linear and $\psi_x,\ \psi_y$ are angular displacement components.
The energy equation of motion is given out by the principle of virtual work,

$$\delta\Pi = \iiint_{vol} \varepsilon^T \delta\sigma \, dV_{ol} - \iiint_{vol} \rho\,\omega^2 \boldsymbol{U} \delta \boldsymbol{U} \, dV_{ol}, \tag{3}$$

Considering the Rayleigh-Ritz method, a set of orthonormal polynomials is chosen as admissible functions shown in the following,

$$u = \sum_{i=1}^{N_u} \sum_{j=1}^{M_u} a_{ij}^u \, \Phi_i^u(x)\, \Psi_j^u(y), \quad v = \sum_{k=1}^{N_v} \sum_{l=1}^{M_v} a_{kl}^v \, \Phi_k^v(x)\, \Psi_l^v(y), \quad w = \sum_{m=1}^{N_w} \sum_{n=1}^{M_w} a_{mn}^w \, \Phi_m^w(x)\, \Psi_n^w(y),$$
$$\psi_x = \sum_{p=1}^{N_{\psi x}} \sum_{q=1}^{M_{\psi x}} a_{pq}^{\psi_x} \, \Phi_p^{\psi_x}(x)\, \Psi_q^{\psi_x}(y), \quad \psi_y = \sum_{r=1}^{N_{\psi y}} \sum_{s=1}^{M_{\psi y}} a_{rs}^{\psi_y} \, \Phi_r^{\psi_y}(x)\, \Psi_s^{\psi_y}(y), \tag{4}$$

where $\Phi_P^S(x)$ and $\Psi_Q^S(y)\,(S = u,\ v,\ w,\ \psi_x,\ \psi_y;\ \ P,\ Q = 1,\ 2,\ 3, \cdots)$ are orthonormal polynomials generated by the Gram-Schmidt process but their first polynomials $\Phi_1^S(x)$ and $\Psi_1^S(y)$ are defined according to the geometric boundary conditions of panel. Substituting equations (4) into equation (3) and the independent property of the coefficients in equation (4) yield the following governing equation,

$$\left| \boldsymbol{K} - \lambda^2 \boldsymbol{M} \right| = \boldsymbol{0}, \tag{5}$$

and then the eigenproblem of the vibration of blades can be solved.

CONCLUSIONS

Using the general shell theory and the first order shear deformation theory, an accurate relationship between strains and displacement components for the Mindlin cylindrical panel with twist and two curvature is derived. The governing equation for free vibration is formulated by the principle of virtual work and the Rayleigh-Ritz method with a set of orthonormal polynomials. Vibration frequencies and modes are achieved for Mindlin cylindrical panels with different geometric parameters such as the twist, the curvature and the aspect ratio, and then the characteristics of the model is revealed.

Acknowledgement

The work described in this paper was fully supported by a grant from the Research Grant Council of the Hong Kong Special Administrative Region [Project No. 7001534 (BC)]

References

[1] RAO J.S.:Natural frequencies of turbine blades–A survey. *Shock Vib. Digest* **5**,3-16,1973.
[2] RAO J.S.:Turbine blading excitation and vibration. *Shock Vib. Digest* **9**,15-22,1977.
[3] RAO J.S.:Turbomachine blade vibration. *Shock Vib. Digest* **12**,19-26,1980.
[4] LEISSA A.W.:Vibration of turbine engine blades by shell analysis. *Shock Vib. Digest* **12**,3-10,1980.
[5] LEISSA A.W.:Vibrational aspects of rotating turbomachinary blades. *Appl. Mech. Rev.* **34**,629-635,1981.
[6] LEISSA A.W.,LEE J.K.,WANG A.J.:Rotating blade vibration analysis using shells. *J. Eng. Power, ASME* **104**,296-302,1982.
[7] LEISSA A.W.,EWING M.S.:Comparing of beam and shell theories for the vibrations of thin turbomachineary blades. *J. Eng. Power, ASME.* **105**,383-392,1983.
[8] LEISSA A.W.,LEE J.K.,WANG A.J.:Vibrations of cantilevered doubly-curved shallow shells. *Int. J. Solids Struct.* **19**,411-424,1983.
[9] LEE J.K.,LEISSA A.W.,WANG A.J.:Vibration of blades with variable thickness and curvature by shell theory. *J. Eng. Gas Turb. Power.* **106**,11-16,1984.
[10] LIM C.W.,LIEW K.M.:Vibration of pretwisted cantilever trapezoidal symmetric laminates. *Acta Mech.* **111**,193-208,1995.
[11] LIM C.W.,LIEW K.M.:The vibration behaviour of shallow conical shells by a global formulation. *Eng. Strucut.* **17**,63-70,1995.
[12] LIM C.W.:Investigating the limit of turbomachinery blade modeling with linear pretwist. In: Ewing, M.S., Leissa, A.W.(Eds.), 2nd Int. Symp. Vib. Continuous Sys.,33-35. Switzerland,12-16 July,1999.
[13] LIM C.W.:Vibration of turbomachinery blades with non-linear twisting curvature. In: Wang, C.M., Lee, K.H., Ang, K.K. (Eds.), APCOM'99,493-498. Singapore, 15-17 December,1999.
[14] TSUIJI T.,TAKAGI K.,YAMASHITA T.:Free vibrations of curved and twisted cantilevered thin plates (Analytical and experimental results). *JSME* **60**(C),4079-4084,1994.
[15] TSUIJI T.,SUEOKA T.:Vibration analysis of twisted thin cylindrical panels by using the Rayleigh-Ritz method. *JSME* **61**(C),4575-4586,1995.
[16] HU X.X.,TSUIJI T.:Free vibration analysis of curved and twisted cylindrical thin panels. *J. Sound Vib.* **219**,63-88,1999.
[17] SAKIYAMA T.,HU X.X.,MATSUDA H.,MORITA C.:Vibration of twisted and curved cylindrical panels with variable thickness. *J. Sound Vib.* **254**,481-502,2002.