SOME NEW THOUGHTS ON THE BUCKLING OF THIN CYLINDRICAL SHELLS

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Summary The classical theory of buckling of axially loaded thin cylindrical shells predicts that the buckling stress is directly proportional to the ratio of thickness to diameter (t/R), other things being equal. But the empirical data show that the buckling stress is proportional to $(t/R)^{1.5}$, other things being equal. Also there is wide scatter in the buckling stress data. The “imperfection-sensitive”, “non-linear” behaviour is thought to be the cause of the above. In this article, it is argued that the boundary conditions, from the “statical determinacy” point of view could explain the shell-buckling paradoxes.

INTRODUCTION

The classical theory, developed in the beginning of the twentieth century, predicts that cylindrical shells will buckle under uniform axial compressive stress s_{cr}, given by

$$s_{cr} = \frac{E}{\sqrt{3\left(1-n^2\right)}}\left(\frac{t}{R}\right) \approx 0.6E\left(\frac{t}{R}\right) \tag{1}$$

where E is the Young’s modulus of elasticity and n the Poisson’s ratio of the isotropic material, t is the thickness and R is the radius of the shell.

However, in the 1930s it was discovered that the experimental buckling loads of cylindrical shells are often small compared to the predictions of the classical theory. Since then there have been extensive research to explain: (i) these lower experimental buckling loads; (ii) the large scatter of the experimental data; and (iii) the catastrophic nature of the failure.

The concepts of “non-linearity” and “imperfection-sensitivity” are widely accepted as explanations for the paradox associated with shell-buckling. However, extensive studies based on precise measurements of geometrical imperfection in physical shells, manufactured in various ways, have had limited success in predicting the buckling load. Therefore, it is reasonable to claim that there must be some crucial factors involved in the mechanisms of shell buckling which are missing from existing analytical studies. This article discusses some interesting features of experiments on the self-weight buckling of small-scale, thin, silicone rubber shells.

SELF-WEIGHT BUCKLING EXPERIMENTS

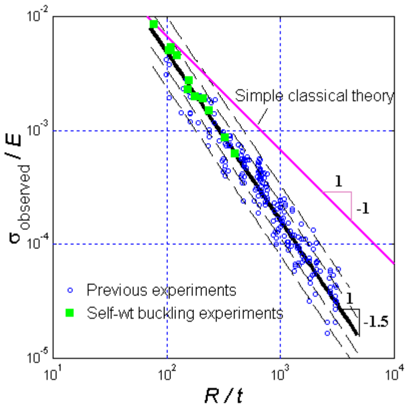


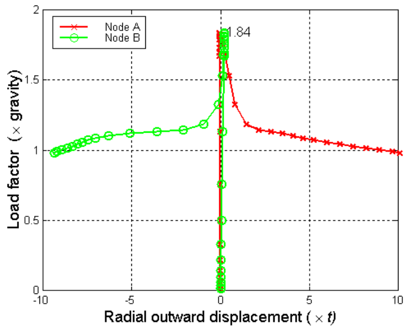
Fig. 1 Log-log plot of experimental data of thin cylindrical shells. The data referred as “previous experiments” have been taken from Brush & Almroth[4]. The heavy best-fit line has a slope of –1.5; and broken lines at 1 and 2 standard deviations from the mean are also shown.

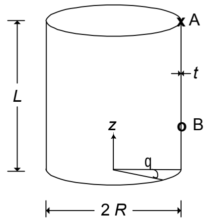
Consider a large-scale, open-topped cylindrical shell. There is a tendency for the structure to collapse under its own weight; and collapse may be precipitated by a disturbance such as an earthquake or wind loading. In order to understand this phenomenon, Calladine and Barber [1] conducted some simple experiments by making long, open-topped, fixed-base thin cylindrical shells in silicone rubber and cutting them down, little by little, until they just stood upright on a flat table without buckling under their own weight. These heights were regarded as the *buckling heights* (L_{cr}) under self-weight. All of the cylinders had the same outside diameter (nominally 172 mm) and ranged in thickness from 0.23 mm to 1.12 mm. The results are shown in Fig. 1, as a log-log plot of dimensionless critical heights against R/t. The ordinate represents the self-weight stress at the base if the shell ($=L_{cr}\ r\ g$, where $r\ g$ is the unit weight of the silicone rubber material), normalized with respect to E. More recently, some more experiments have been conducted with a larger mould, of diameter 241 mm: see Refs.[2] and [3]. These results are also plotted in Fig. 1, and they agree very well with [1]. There are two distinguishing features of these results. First, in general the buckling clearly does not take place according to the “simple classical theory” (Eqn 1). Second, there is very little scatter in the data, compared to almost all buckling tests on thin cylinders that have been reported in the literature. To assess the situation, the data of experimental buckling strength of cylindrical shells under axial compression that was collected and presented by Brush and Almroth [4], are re-plotted here in Fig 1, on log-log scales of s_{cr}/E and R/t. It is immediately clear that the experimental data lie on a band which is steeper than the “classical line”; and indeed the best-fitting line through the data points has a slope of –1.5, to two significant figures. Specifically, we find

$$s_{mean} = 5\,E\left(\frac{t}{R}\right)^{1.5} \tag{2}$$

Non-linear finite element analyses

In order to understand the paradoxical self-weight buckling, non-linear finite element analyses of several experimental shells were conducted using ABAQUS [5]. The result of a typical computation is shown in Fig. 2. The figure shows that the shell does not buckle until a load of 1.84*g* has been reached: this is near to the classical buckling load level for the shell as revealed by a linear-eigenvalue buckling analysis. The load then falls to around 1.3*g*, with very little radial displacement. After this, a dimple starts to appear at Node B, with significant inward displacement, and Node A at the top starts to move outwards. The shell above the dimple deforms inextensionally henceforth, leading to a post-buckling "plateau" corresponding to the experimental critical heights. The "plateau" remains virtually similar if the amplitudes of the initial imperfection were increased by a factor of four.





Imperfection used in the ABAQUS computation:

$$-0.25\, t\,(z/L)\cos(7q)\,e^{-6.9q^2}$$

Fig. 2 Plot of numerical gravity load factor against radial deflection at two points of an open-topped, fixed base shell ($R/t \approx 210$) from the self-weight buckling experiments. An ordinate of value 1.0 corresponds to the physical shell under gravity at its measured critical height. The geometrical imperfection was deliberately included, primarily as a requirement for the non-linear analysis to be performed smoothly; and the shape of the imperfection was chosen so as to encourage buckling to begin at a desired location.

DISCUSSION AND CONCLUSIONS

The self-weight buckling experiments along with the non-linear finite-element analyses highlight that the critical stress of cylindrical shells under axial compression is proportional to $(t/R)^{1.5}$, compared to $(t/R)^{1.0}$ as in classical theory. This fits well with the large amount of previous experimental data. However, the experiments on self-weight buckling showed negligible scatter compared to the earlier experiments in the literature. We suggest that the explanation lies in the fact that the two sets of shells have a different status in terms of *static determinacy*: thus the open-topped shells in the self-weight buckling experiments are *statically determinate*, whereas almost all the shells used in standard buckling tests in the past had closed ends, hence they are *statically indeterminate* from the membrane stresses point-of-view. The static indeterminacy in a system may be held responsible for any locked-in stresses due to lack-of-fit resulting from the manufacturing process. Such unaccounted, random locked-in-stresses may cause a shell to buckle at a premature load, after reaching the critical stress in a localised area; or in general they may produce a big statistical variation in buckling loads of notionally similar shells.

To verify the above hypothesis of *static determinacy,* another set of non-linear finite element analyses were conducted by Vossos [6]. Three fixed-base cylindrical shells were considered. The boundary conditions at the top were: (i) open, (ii) constrained to be both circular and plane, and (iii) attached to a ring beam. The self-weight buckling analyses of these three shells were carried out in two steps. In the first step a preload was applied to mimic the "residual stresses" in practice, and in the second step gravity load was applied by *modified RIKS* method in ABAQUS [5]. The preload was applied in one of the following ways: (i) a concentrated load in the radial inward direction, applied near Node B (see Fig. 2); (ii) a temperature increase at a small region near Node B; and (iii) uplift of the base at a small portion of the perimeter of the shell. The comparative buckling load factors are shown in Table 1. The buckling load, of course, depends on the amount of preload and their positions. Hence the values given in Table 1 should be read column-wise. They do not have any meaning along a row.

It is clear that the statically determinate shells perform better in buckling than their statically indeterminate counterparts. For example, an increase in temperature in a small region of a statically indeterminate shell will cause higher initial stresses. As a result, when the shells are tested for buckling, the statically indeterminate shells will carry lower loads.

	Boundary Conditions		Statical	Buckling load factor		
	Top	Base	Determinacy	Radial load	Temperature	Uplift
Shell 1	Open	Fixed	Determinate	1.00	1.00	1.00
Shell 2	Circular & Plane	Fixed	Indeterminate	0.90	0.66	0.41
Shell 3	Ring beam	Fixed	Indeterminate	0.94	0.57	0.49

Table 1

References

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