YIELD SURFACES USING AN EXTENSION OF THE "REGULARIZED" SCHMID LAW TO POLYCRYSTALLINE MATERIALS

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Summary A new micromechanical model based on an extension of a "regularized" Schmid law to polycrystalline materials is presented. The homogenization procedure is based on the Transformation Field Analysis method (TFA). Yield surfaces for FCC metals are generated .

INTRODUCTION

On the mechanical ground, crystal plasticity obeys a non linear and anisotropic flow law, which has been differently described according to the chosen assumption of rate-independent or rate-dependent plasticity. In the former case, the Schmid flow criterion is used, while in the latter, power law descriptions are widely called for. More recently, a "regularized" Schmid flow rule has been proposed [1,2] to eliminate the singularities of the classical Schmid yield surface for single crystals. In this work, the "regularized" Schmid flow law is considered to account for a heterogeneous plasticity in a FCC polycrystal. Being so treated as an aggregate of plastically flowing grains, the effective polycrystal behavior, and the stresses in each grain, can be estimated by using the Transformation Field Analysis (TFA), which is an elastic "affine" homogenization procedure [3]. The paper is organized as follows. The first part introduces the micromechanical model in the framework of a "regularized" Schmid law and details the homogenization step. The second part gives numerical results and particularly yield surfaces obtained for isotropic FCC polycrystals. Concluding remarks are given in the last part.

MICROMECHANICAL MODEL

Regularized Schmid law and crystallographic approach

The yield criterion proposed is an extension to an aggregate of the "regularized" Schmid law proposed initially by Gambin [1] and Arminjon [2] for single crystals. In this work, the plastic flow criterion reads:

$$F = \left(\sum_{I} f^{(I)} \sum_{g(I)} \left(\frac{\left| \tau^{g(I)} \right|}{\tau_c^{g(I)}} \right)^n \right)^{\frac{1}{n}} - 1 = 0 , \qquad (1)$$

where n is an exponent which has to be correctly chosen. $\tau_c^{g(I)}$ deals with the strain-hardening state and $\tau^{g(I)}$ is the resolved shear stress on each slip system g(I) for the set of grains I with volume fraction $f^{(I)}$. For each grain, a homogeneous multiple (anisotropic) slip description is used and the set of slip systems {111} <110> is considered. By calculating the differential form of F and accounting for the whole microstructure of the polycrystal, the equation $dF = 0$ is solved and leads to the definition of a new plastic multiplier $d\lambda$. Then, the plastic slip increment is defined by:

$$d\gamma^{g(I)} = d\lambda \frac{\partial F}{\partial \tau^{g(I)}} . \qquad (2)$$

The form of (1) is convenient with most of the FEM codes and includes in the elastic-plastic moduli the crystallographic nature of the material, in particular its textural anisotropy and its intra crystalline strain-hardening.

Homogenization procedure

The framework of the so called "affine formulation" [4] has been used and more specifically the TFA procedure proposed by Dvorak [3] which implies only concentration stress and strain tensors calculated in elasticity. This elastic nature within the TFA method gives a useful simplicity to model strongly non linear behaviors like elastic-plastic polycrystals. Thus, the resolved shear stress takes the following form:

$$\tau^{g(I)} = R^{g(I)} : \sigma^{(I)} = R^{g(I)} : \left(B^{(I)} : \Sigma + \sigma_0^{(I)} \right), \text{ and:} \qquad (3)$$

$$\sigma_0^{(I)} = \sum_{J} G^{(IJ)} \varepsilon^{p(J)} , \qquad (4)$$

where $R^{g(I)}$ is the orientation tensor, $\sigma^{(I)}$ is the local stress tensor and $G^{(IJ)}$ is a related to the elastic stress concentration tensor $B^{(I)}$, the elastic properties and the volume fraction of each grain J; $B^{(I)}$ is calculated using the

self-consistent scheme in elasticity. The effective plastic strain of the Representative Volume Element (RVE) is given by:

$$E^{Pe} = \sum_{I} f^{(I)}\ {}^{t}B^{(I)} : \varepsilon^{p(I)} . \qquad (5)$$

Results are provided considering a RVE of spherical grains characterized by their Euler angles and their volume fraction.

NUMERICAL RESULTS

Single crystal

Following the work of Franciosi et al. [5] for FCC metals, the hardening matrix is composed of 2 terms. The first term corresponding to self-hardening is H1=μ/250 (slope of stage II for a tensile test on FCC single crystal [4] where μ is the shear modulus); the second term corresponding to latent hardening is H2=qH1 with q=1.4. Tensile curves for different grain orientations capture the marked orientation dependence for FCC single crystals. Yield surfaces (obtained with $f^{(I)}$=1 and 1 grain) for different orientations are in agreement with previous works [1,2]. Elastic constants for Al are used in the simulations and $\tau_0 = 2$MPa for each slip systems.

Yield surfaces for polycrystals

Macroscopic tensile curves denote a linear stage consistent with a slope of E/100 (E: Young modulus) characterizing FCC polycrystalline metals. Yield surfaces have been successfully generated for isotropic (Fig. 1) and textured polycrystals. For n>100, the shape of the initial yield surface is in agreement with the one obtained by the classical Schmid law. For n=2, yield surfaces are similar to Hill's 1948 and Von Mises quadratic criterions respectively for anisotropic and isotropic aggregates.

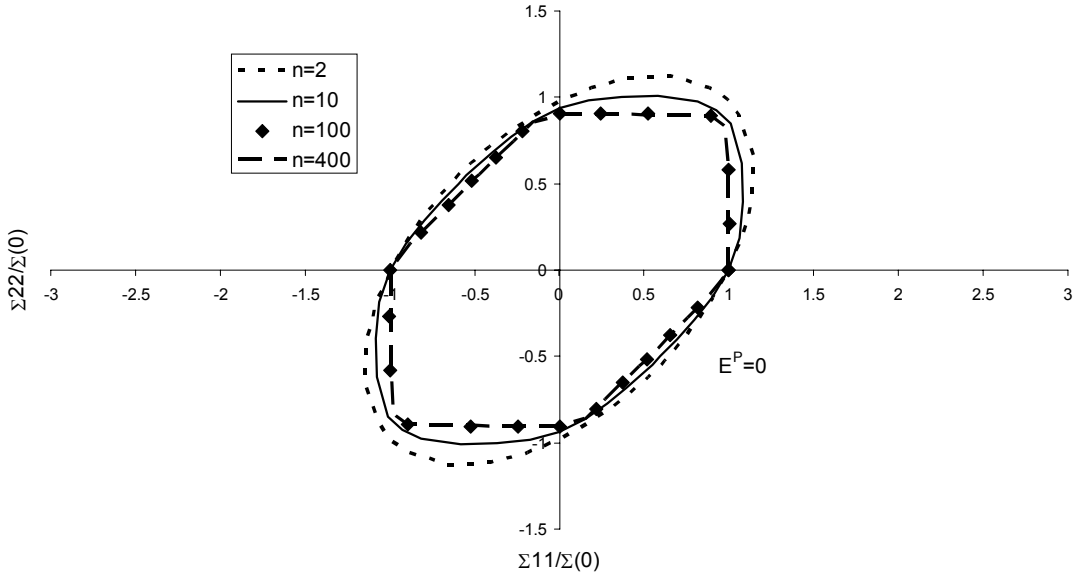

Fig.1. Normalized (initial) yield surfaces of an isotropic aggregate (2016 orientations) for several values of n.

CONCLUSIONS

The present model based on a "regularized" plastic flow rule and the TFA approach avoids the ambiguity problem for the selection of active slip systems within elastic-plastic polycrystals. Yield surfaces are in good agreement with the ones obtained using the classical Schmid law. At this step, a hypothesis of "average" plastic strain per crystal is used. We propose to subdivide each crystal into a N-laminate structure, according to the N slip planes of the considered lattice type, such that the plastic strain fields are piecewise uniform. That allows to use the TFA method at the micro level to better describe the intra granular deformation mechanisms and to estimate more accurately the overall behavior of polycrystals.

References

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