THE ESHELBY PROBLEM FOR ELASTIC VISCOPLASTIC MATERIALS

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Summary Micro-macro schemes contain two important steps. First, the localization process must link the fields inside the inclusion to those applied in the infinite matrix. Secondly, an averaging process is used to derive the macroscopic behavior. For elastic viscoplastic materials, there is no exact solution for the first step. In this work, we adopt the interaction law proposed in [2]. By comparison with Finite element calculations, it is shown that the localization step is well described by the proposed interaction law.

THEORY

To account for intergranular heterogeneities during deformation of elastic-viscoplastic polycrystals, one has to face complicated problems due to the constitutive behavior of the phases. The global elastic viscoplastic behavior is characterized by a long memory effect. Attempts have been proposed recently, based on the extension of interaction laws developed for elastic-plastic materials. These approaches overestimated the internal stresses. In other approaches, the elastic-viscoplastic behavior is linearized at each step of the deformation and this linear problem is solved with Laplace transform. These techniques are complex and time consuming. A different way has been proposed by Kouddane et al in [1] and extended by Molinari [2].
In this work, the Eshelby problem is considered, where an ellipsoidal inclusion is embedded in an infinite homogeneous matrix. Homogeneous strain rate is applied at the remote boundary of the matrix. A small deformation theory is adopted so that the total strain rate is the sum of the elastic and viscoplastic strain rates :

$$\underline{d} = \underline{d}^{e} + \underline{d}^{vp}$$

Both phases (inclusion and matrix) have non linear elastic-viscoplastic behaviors. The rate of the Cauchy stress $\underline{\dot{\sigma}}$ is linked to the elastic strain rate as follows :

$$\underline{\dot{\sigma}} = \underline{\underline{a}}^{e} : \underline{d}^{e}$$

The viscoplastic response is described by a powerlaw but with no restriction, other type of constitutive responses can be analysed. The viscoplastic strain rate is linked to the deviatoric Cauchy stress $\underline{s}$ by :

$$\underline{d}^{vp} = \frac{3}{2}\frac{\sigma^{eff}}{d^{eff}}\underline{s} \qquad \text{with} \qquad \sigma^{eff} = k(d^{eff})^{m}$$

σ^{eff} and d^{eff} are the effective stress and strain rate. m denotes the strain rate sensitivity, k the hardness.
The proposed interaction law [2] is :

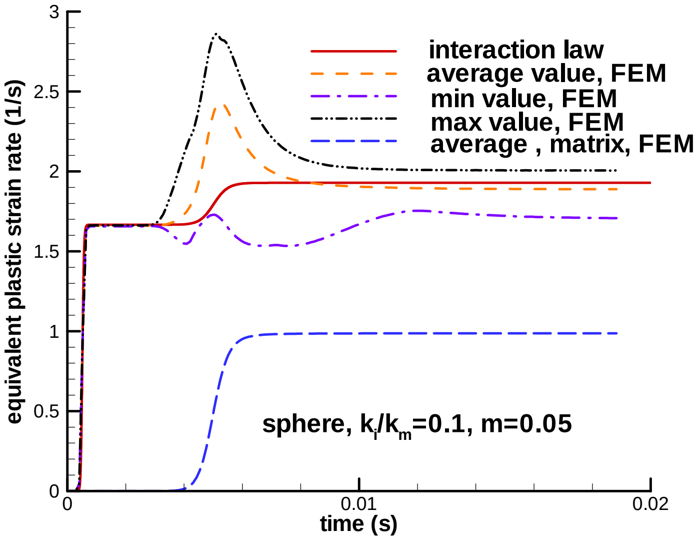
$$\underline{d}^{i} - \underline{D} = (\underline{\underline{A}}^{tg} - \underline{\underline{P}}^{tg^{-1}})^{-1} : (\underline{s}^{i} - \underline{S}) + (\underline{\underline{A}}^{e} - \underline{\underline{P}}^{e^{-1}})^{-1} : (\underline{\dot{\sigma}}^{i} - \underline{\dot{\Sigma}})$$

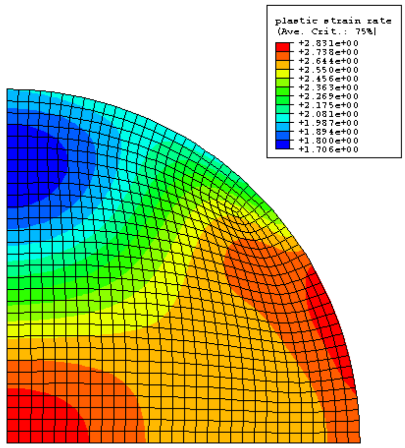
$\underline{d}^{i}, \underline{\sigma}^{i}, \underline{s}^{i}$ correspond to the average values in the inclusion. $\underline{D}, \underline{\Sigma}, \underline{S}$ are the strain rate and stress fields prescribed in the matrix far from the inclusion. $\underline{\underline{A}}^{tg}$ and $\underline{\underline{A}}^{e}$ are the viscoplastic tangent moduli (depending on the strain rate $\underline{D}$) and the elastic moduli of the matrix. $\underline{\underline{P}}^{tg}$ (resp. $\underline{\underline{P}}^{e}$) is a fourth order tensor calculated with use of Green functions related to $\underline{\underline{A}}^{tg}$ (resp. $\underline{\underline{A}}^{e}$).

RESULTS

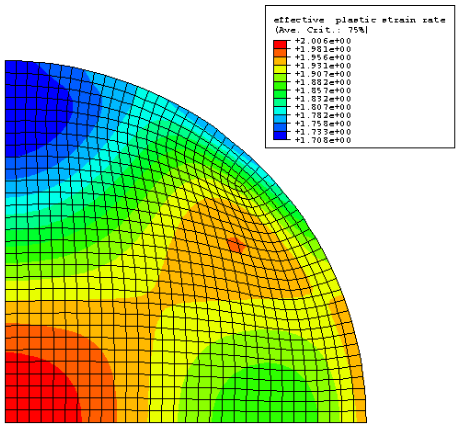
A comparison between predictions of the interaction law and finite element calculations performed with ABAQUS software is carried out. The goal of this study is to demonstrate the capability of the interaction law to capture the elastic-viscoplastic coupling.
The applied loading prescribed at the remote boundary is the following $D_{11} = 1s^{-1}, D_{22} = -0.5s^{-1}, D_{33} = -0.5s^{-1}$.
For incompressible (elastic and viscoplastic) and linear behavior ($m = 1$), the time evolving solution for the strain rate field in the inclusion can be found analytically using Laplace transform. The prediction of the interaction law coincides with the analytical solutions. For non linear elastic-viscoplastic bahavior, no exact solution exists. Numerous F.E. calculations have been performed for different shape of inclusion (sphere , prolate or oblate ellipsoids), hardness ratio, strain rate sensitivity and elastic properties. It can be observed that in all cases (compressible or incompressible elasticity) the average value of the strain rate field inside the inclusion is in close agreement with the average value obtained with use of ABAQUS. As can be seen on the figure, the elastic response at the beginning of the loading is accurately captured. Latter in the deformation process, the elastic contribution vanishes and the fully viscoplastic solution is retrieved. The validity of the interaction law in viscoplasticity (with no elastic deformation) has been demonstrated in [3]. Therefore the proposed interaction law is able to capture accurately the instantaneous elastic

response as well as the asymptotic viscoplastic state. For intermediate time, where the elastic-viscoplastic coupling occurs, the results derived from the interaction law follows the trends obtained with ABAQUS. Note the heterogeneity inside the inclusion occuring during the loading is clearly observed with the F.E. The proposed interaction law can only provide information concerning the average value.





t=5.2e-3 s



t=1.8e-2 s

Figure : Comparison between prediction of the interaction law and F.E. calculations. The inclusion has a spherical shape. The hardness ratio is k_i / k_m =0.1. The matrix and the inclusion have the same strain rate sensitivity m=0.05 and the same incompressible elastic behavior ($\mu_i = \mu_m = 66.6$). On the left, the plastic strain rate heterogeneity in the inside the inclusion is shown (results obtained with ABAQUS).

Note that comparisons have been performed also for lower remote strain rate $D_{11} = 10^{-4}\ s^{-1}$ and for cyclic loading. It is observed that the interaction law is able to describe correctly the elastic-viscoplastic coupling.
Using a self consistent scheme together with the interaction law, which is now validated, the macroscopic behavior of a two phases material has been described. When the two phases have linear incompressible behavior, an analytical solution exists. Good predictions are obtained by the proposed model.

CONCLUSIONS

The localization step for elastic-viscoplastic behavior is well captured by the interaction law proposed in [2]. Finally, this interaction law is used with an averaging scheme to obtain the macroscopic behavior of multiphase elastic viscoplastic materials.

References

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