STABILITY ANALYSIS OF A TETHERED SYSTEM

J. Valverde, J.L. Escalona and J. Domínguez
Dept. of Mechanical and Materials Engineering, University of Seville, Spain

Summary The SET is an electrodynamic tether (thin rod) that is subjected to a continuous rotation about the longitudinal axis. The system is similar to an umbalanced rotor. The linear rotordynamics theory predicts instability above a critical velocity. However, it has been probed that the geometric non-linearities can stabilize the system under certain operating conditions.

Space-tethers are very slender and flexible structures that are deployed in outer space. In recent times space-tethers have been the subject of numerous studies in the field of astronautics. The material presented here continues the work carried out by Valverde et al. [1] on a prototype of the European Space Agency called SET (Short Electrodynamic Tether). The SET comprises a central module or main satellite from which two tethers extend. The tethers are about 100 m long and have a tiny cross-section of about $4\cdot 10^{-6}$ m^2 (thin rod), see Fig. 1. The tethers are electric conductors so the SET interacts with Earth's magnetic field. A plasma contactor (modelled as a disk) is placed at the end of each tether. Due to the Faraday effect [2], a thrust or drag force can be created without spending chemical fuel. Hence, the satellite can be maneuvered without cost.

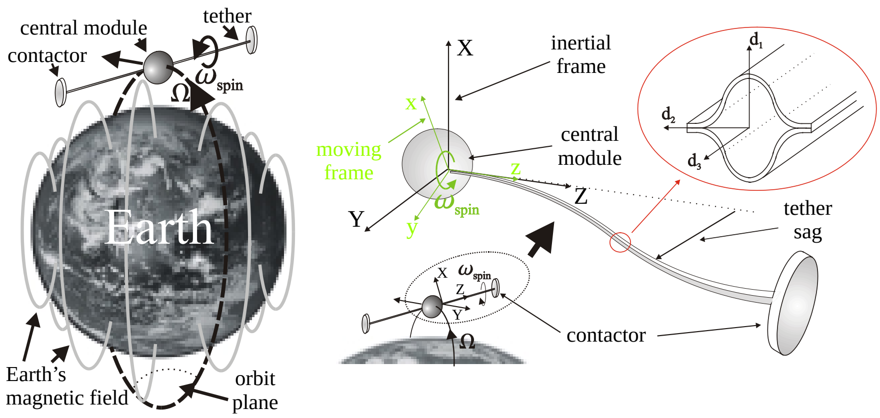

Figure 1. SET orbit and model.

Typically, electrodynamic tethers orbit in the natural position of a free body in space, that is to say, with the axis of smallest moment of inertia pointing towards the centre of the Earth. In this position (vertical tether), it is structurally stable due to the gravity gradient [2]. At high ionosphere altitudes (low orbits, 50-100 *km*), the usual vertical tethers are highly inefficient due to the shape of the Earth's magnetic field. In this case, the axis of the smallest moment of inertia needs to be perpendicular (normal position) to the orbit plane in order to interact in a convenient way with the magnetic field lines, see Fig. 1. Due to the gravity gradient, the system, however, has a tendency to migrate so that it is pointing towards the centre of the Earth. In order to avoid this, a spin velocity (ω_{spin}) is applied about the axis of smallest moment of inertia and the gyroscopic pairs maintain the system in the desired operating position. However, the presence of a gravity gradient does not guarantee the structural stability of the tethers in this normal position.

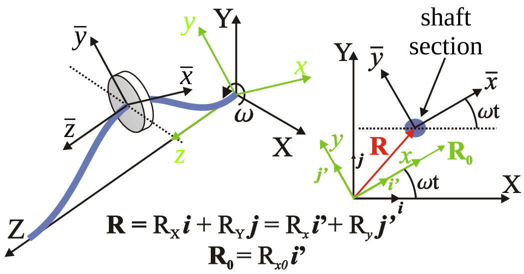

Figure 2. Jeffcott model.

The tethers are stored rolled up in drums prior to extension. The storage and deployment of the tethers from the central module produces an initial deviation from the straight configuration, or sag, when extended into their operating position, i.e., the tether is intrinsically curved when it is deployed, see Fig. 1. As can be seen in Fig. 1, the tether cross-section is orthotropic. The aim of this work is to study the dynamics of the system and to analyze the conditions (values of ω_{spin}, etc.) for which the described operating position is stable. As the spin velocity of the SET is much higher than the rotational velocity about the Earth Ω, a coordinate frame {X,Y,Z} located at the central module can be considered inertial, see Fig. 1.
In order to understand the problems related to the structural stability of the SET, a simple low-degree-of-freedom unbalanced rotor-system analogy can be used. Fig. 2 shows the Jeffcott model which has been successfully used in rotordynamics to characterize the different problems that arise in rotating devices such as engine rotors,

turbomachinery, etc. The main feature is that this easy-to-manage system is suitable for the study of different phenomena such as the bearing clearances in rotor supports, external damping, etc. The Jeffcott model is a three degrees of freedom system which contains the main dynamic properties of a rotor system. If only one of the tether-contactor systems is taken into account (assume symmetry with respect to the orbit plane, see Fig. 1), the tether is analogous to the shaft and the contactor is analogous to the rotor [3]. If the motion of the shaft-disk model is asynchronous, alternative stresses arise in the shaft and internal damping forces appear in the system due to the hysteretic dissipative process. In this situation, for spin velocities above the first bending natural frequency of the system (critical velocity), the linear rotordynamics theory predicts structural instability in the shaft. Since the motion of the tether-contactor system is asynchronous due to the torsion in the tether and the spin velocity of the SET is higher than the critical velocity, it seems that the SET would be structurally unstable. The tether sag would increase without bound and the SET would not work conveniently.

Previous works on rotordynamics [3] suggest that a rotor system with internal damping forces is stable due to the effect of geometric non-linear elastic and internal damping forces. In order to probe the previous assertion, the authors studied [1] the fully non-linear system by means of multibody dynamic simulations. These dynamic simulations show that the tether sag could remain bounded under certain operating conditions. It would be useful to know under which values of the spin velocity, initial deviation, etc. the SET behaves as a structurally stable system.

In this work, a stability analysis of a non-linear simplified system has been carried out. In particular, equilibrium or periodic solutions of the rotating system are sought under different conditions. Once the steady-state conditions of the system are obtained as a function of a set of parameters, namely ω_{spin}, length, initial deviation, etc., the stability of these solutions is studied. Therefore, a general map regarding the dynamics of the SET can be outlined and the behaviour of the system can be finally understood. It is important to take into account that the non-linearity of the system plays an essential role in the behaviour of the SET. The geometric non-linear terms completely change the dynamic behaviour (topology) of the system, in a way that a linear model of the SET is structurally unstable above the critical velocity but a non-linear model has been probed to be stable under certain operating conditions [1]. A modified Jeffcott model with geometric non-linear elastic and damping forces is simple enough to develop the stability analysis analytically. An equivalent viscous damping model has been used to describe the internal hysteretic damping of the material. The power dissipated by the damping forces is obtained and used as a Rayleigh dissipation function. The shaft has been discretized by means of Rayleigh-Ritz method. One shape function has been used in each transversal direction (*x, y*). The system is studied in moving frames {*x,y,z*} instead of inertial frames because the expected periodic solutions are equilibrium points in moving frames, easier to manage. Two different cases have been studied: isotropic case (circular cross section shaft) and orthotropic case (real tether cross section). The initial deviation is assumed to appear in the direction of minimum second moment of area of the section, R_{x0}. For subcritical velocities, the system is stable as the linear rotordynamics theory predicts. On the other hand, for supercritical velocities (operating conditions of the SET) different features arise. In both cases, a branch of equilibrium solutions in the initial deviation *x*-plane appear ($R_y = 0$). These solutions are given by the following cubic equation ("in-plane solutions"):

$$y = (R_x - R_{x0}) \quad \therefore \quad y^3 + (1-\varpi^2)y = \varpi^2 R_{x0} \;\Rightarrow\; f(y,\varpi) = \lambda(R_{x0},\varpi) \;\Rightarrow\; \bar{R}_x \ equilibrium$$

where ϖ is the dimensionless angular velocity. The stability of these solutions is shown in Fig. 3. Depending on the value of the parameters, there can be 1, 2 or 3 equilibria. In the orthotropic case, there is also a branch of solutions ("out-of-plane solutions") out of the initial deviation plane ($R_y \neq 0$). In any case, there is always one stable equilibrium (periodic in inertial frames) to which the system finally tends.

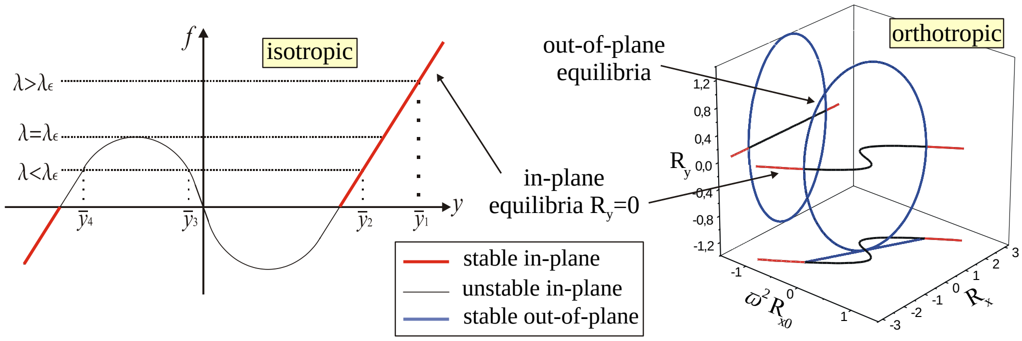

Figure 3. Jeffcott model equilibria. Isotropic and orthotropic cases.

Qualitatively, this stability analysis supports the results obtained from the dynamic simulations [1].

References

[1] Valverde, J., Escalona, J.L., Mayo, J. and Domínguez, J., "Dynamic analysis of a light structure in outer space: Short Electrodynamic Tether", *Multibody System Dynamics* **10**(1) 125-146, 2003.

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[3] Childs, D.W., *Turbomachinery Rotordynamics*, Wiley Interscience, New York, 1993.