EFFICIENT GENERALIZED SPEEDS IN A RECURSIVE FORMULATION OF FLEXIBLE MULTIBODY DYNAMICS

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Summary
A recursive formulation of the dynamical equations of an articulated multibody system with flexible components is given in terms of efficient motion variables for elastic motion and hinge rotation about revolute, Hooke and spherical joints. We treat motion under external forces, as well as prescribed motion with internal loads calculation, and show computational efficiency of the formulation with examples.

Introduction

Simulations of large overall motion of a hinge-connected multibody system with elastic components tend to be computationally intensive. Recently, Banerjee [1] reviewed methods of reducing computer time that include defining new motion variables and using recursive or parallel algorithms. Mitiguy and Kane [2] had given a choice of generalized speeds that reduce simulation time for a system of rigid bodies with rotation in revolute, Hooke, and revolute joints. D'Eleuterio and Barfoot [3] proposed a variable for describing elastic motion that yields a constant mass matrix for a single flexible body in large overall motion. This paper uses these motion variables to modify the recursive formulation of Ref. [4] for a system of hinge-connected flexible bodies. The formulation is also extended to situations where some or all of the degrees of freedom at a joint are prescribed, as when determining internal loads. Examples demonstrate the computational efficiency of the formulation.

Single Flexible Body in Large Overall Motion

Using generalized speeds u_i representing orthogonal components of angular velocity of a reference frame and the velocity of a point fixed in that frame, with vibration mode variables of Ref. [3], the velocity of a material point and the kinematical equations for n modal coordinates can be shown to be

$$\mathbf{v} = \sum u_i \mathbf{b}_i + \sum u_{3+i} \mathbf{b}_i \times \mathbf{p} + \sum \boldsymbol{\varphi}_i u_{6+i} \tag{1}$$

$$\sum \int \boldsymbol{\varphi}_i \bullet \boldsymbol{\varphi}_j \, dm \, (u_{6+j} - \dot{q}_j) = \sum u_{3+i} \mathbf{b}_i \bullet \sum \int \boldsymbol{\varphi}_j \times \boldsymbol{\varphi}_i \, dm \, q_j , \quad (i, j = 1,\dots,n) \tag{2}$$

Kane's dynamical equations [5] for a body with potential energy P and dissipation function D are

$$\int \frac{d\mathbf{v}}{dt} \bullet \frac{\partial \mathbf{v}}{\partial u_i} dm = - \sum [\frac{\partial P}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j}] \frac{\partial \dot{q}_j}{\partial u_i} , \qquad (i = 1,\dots,6+n) \tag{3}$$

Eqs. (3) lead to the same equations as those obtained in Ref. [3] by using Hamilton's principle with quasicoordinates. Eqs. (1) and (3) reveal that the mass matrix in Eqs. (3) is time-invariant. Rotation-deformation coupling in Eqs. (2) produces contributions to generalized active force due to elasticity, geometric stiffening, and dissipation in Eqs. (3) for rotational as well as modal generalized speeds.

Kinematics of Hinge Motion in Flexible Multibody Systems

Angular velocity $\boldsymbol{\omega}^j$ of the reference frame of body j connected to inboard flexible body c(j) is written after accounting for elastic rotation rate at the hinge of body c(j) in its reference frame. Following Ref. [2] for a Hooke's joint, with generalized speeds u_1 and u_2, hinge vectors $\mathbf{h}_1$and $\mathbf{h}_2$, and unity dyadic $\mathbf{U}$, this yields

$$\boldsymbol{\omega}^{j} = [\boldsymbol{\omega}^{c(j)} + \sum \varphi_i^{c(j)} \dot{q}_i^{c(j)}] \bullet [\mathbf{U} - \mathbf{h}_1 \mathbf{h}_1 - \mathbf{h}_2 \mathbf{h}_2] + u_1 \mathbf{h}_1 + u_2 \mathbf{h}_2 \tag{4}$$

Revolute joint kinematics is a reduction of Eq. (4). Generalized speeds for a spherical joint are body components of angular velocity and those for slider joints are relative translational rates. These generalized speeds require modifying the kinematics in the algorithm of Ref. [4], with the dynamical equations generated in a sequence of forward, backward, and forward passes. When a generalized speed at a hinge is prescribed, the associated internal load is obtained by dot-multiplying the augmented inertia and active torques and forces by the partial angular velocity or partial velocity [5] for the prescribed joint motion.

Simulation Results

Large angle slewing of a single flexible body, a solar sail spacecraft with 44 elastic modes, is simulated by the above formulation and compared to Standard Kane and recursive methods using customary generalized speeds (time derivatives of translational, rotational and modal generalized coordinates). Identical results as in Figs. 1 and 2 are obtained for this 50 degrees of freedom (dof) system for a 300 sec simulation.

Method:	**Standard Kane**	**Recursive (Customary Variables)**	**Recursive (Efficient Variables)**
CPU Times:	**699 sec**	**667 sec**	**140 sec**

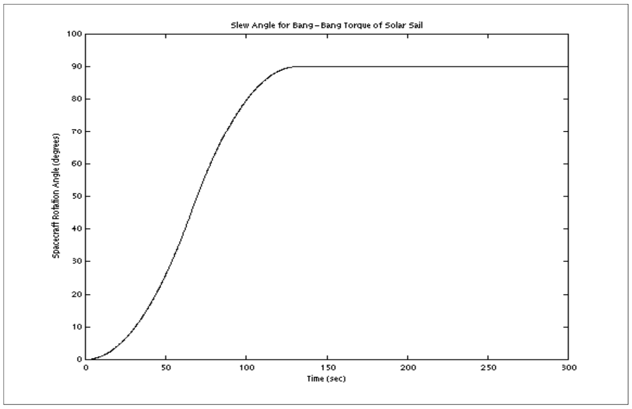

Fig. 1 Solar sail large angle slewing

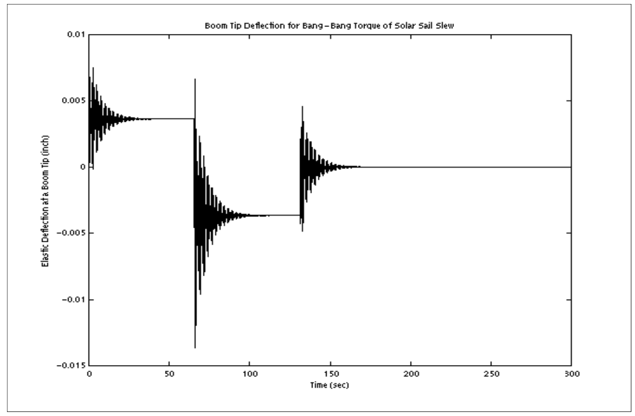

Fig. 2 Sail boom tip deflection with bang-bang torque

Figures 3 shows four flexible bodies connected by three types of rotational joints, and Figure 4 overlays plots of an internal load computed by the three methods. A 10 sec simulation of this articulated 52 dof system requires the following computation times.

Method:	**Standard Kane**	**Recursive (Customary Variables)**	**Recursive (Efficient Variables)**
CPU Times:	**10.2 sec**	**7.6 sec**	**5.2 sec**

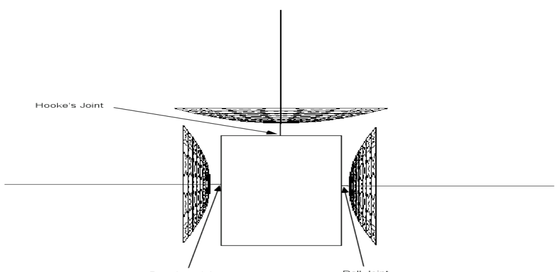

Fig. 3 Four flexible bodies with revolute, Hooke, and ball joints

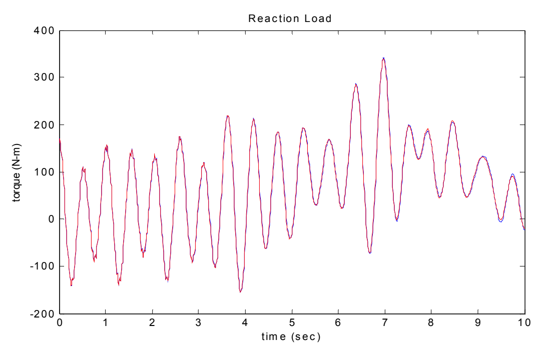

Fig. 4 Load for a prescribed rotation

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