THE STRUCTURE OF CONSTITUTIVE LAWS FOR POWDER METALLURGICAL COMPONENTS

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Summary Deformation and incremental theory models of the deformation of particulate materials are presented. A potential surface can be constructed for the deformation theory by determining the complementary work done along extremal paths in stress space. Yield surfaces for the incremental model nest inside these surfaces.

INTRODUCTION

In this paper we examine the structure of constitutive laws for the compaction response of particulate materials, where deformation results from the plastic deformation of the individual particles. Micromechanical modelling suggests a number of different structures for the constitutive laws. A deformation theory model is described which is based on the construction of surfaces of constant complementary work density in Kirchhoff stress space. Alternatively, an incremental anisotropic constitutive law can be constructed based on the existence of a yield surface, whose size and shape evolves as the powder compact is deformed plastically. It is demonstrated that the yield surfaces nest inside surfaces of constant complementary work density. The models can be calibrated using data generated in a triaxial cell. The nesting character of the different types of surface is demonstrated for a commercial steel powder.

THEORETICAL CONSIDERATIONS

Ponter and Martin [1] examined the relationship between incremental and deformation theories of plasticity and identified a set of extremum theorems which allow an appropriate deformation theory to be developed for a given assumed incremental theory. They demonstrate that fora given incremental plasticity model that there is exists a path, or paths, in stress space which maximises the complementary work $\overline{\varpi}(\sigma_{ij})$, ie

$$\overline{\varpi}_e(\sigma_{ij}) \geq \overline{\varpi}(\sigma_{ij}) \qquad (1)$$

The strain accumulated along the extremal path is then given by

$$\varepsilon_{ij} = \frac{\partial \overline{\varpi}_e}{\partial \sigma_{ij}} \qquad (2)$$

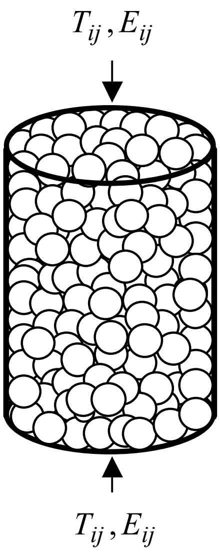

Figure 1 Random arrangement of spherical particles

The extremal paths effectively define a stable hyperelastic material which is less stiff than the inelastic material used to define it. Now consider a porous or particulate body which is made of the hyperelastic material, as shown in Figure 1, where T_{ij} represents the Kirchhoff stress and E_{ij} the true strain. It can be shown that

$$E_{ij} = \frac{\partial \overline{\Omega}_e}{\partial T_{ij}} \qquad (3)$$

where $\overline{\Omega}_e = \frac{1}{V}\int_V \overline{\varpi}_e dV$. It is possible to determine lower bounds to the potential $\overline{\Omega}_e$ in terms of any assumed internal starin field that is compatible with the macroscopic strain E_{ij} [2]. It can be further shown that if the incremental plasticity material is rigid-perfectly plastic, that the macroscopic yield surfaces for states generated from macroscopic extremal paths nest inside the surface of constant $\overline{\Omega}_e$. This is illustrated in Figure 2, where the yield surfaces and the surface of constant $\overline{\Omega}_e$ were constructed using the same material and geometric assumptions as Fleck [3].

EXPERIMENTAL STUDIES

Experimental studies have been constructed on a range of metal powders with particles of different size, shape and composition using a computer controlled triaxial cell [4] in which cylindrical samples are subjected to axial and radial stresses. Specimens have been loaded along either radial loading paths in stress or strain space to a prescribed level of complementary work per unit current volume (ie constant $\overline{\Omega}_e$). The specimens were then unloaded and the yield surface corresponding to the state created from each of the initial radial loading paths was determined by a probing technique in which the specimen was reloaded along a series of different paths until plastic strain was detected (further details are given in [4]). Figure 3 shows a surface of constant $\overline{\Omega}_e$ determined during the initial loading (the dashed curve) and the yield surfaces corresponding to a series of different initial loading paths (coloured solid curves) for a commercial steel powder. The yield surfaces nest inside the surface of constant $\overline{\Omega}_e$. Also the strain corresponding to a given loading path is normal to the surface of constant $\overline{\Omega}_e$, in agreement with eqn (3) [5]. As the ratio of deviatoric to mean stress increases the yield surface becomes more elongated in the direction of loading, in agreement with the micromechanical model. Experimentally, however, these surfaces can be approximated as ellipses. As the compact is deformed further to higher levels of stress and strain the elliptic yield surfaces grow and their aspect ratio decreases, until at high relative densities they virtually fill the surface of constant $\overline{\Omega}_e$, ie the yield surface is not very sensitive to the detailed loading path. Schneider [4] has described a constitutive model for the evolving yield surface which describes the evolution of the size, aspect ratio and orientation in terms of the accumulated plastic strain.

The deformation theory model uses the initial dense random packed state of the particles as the reference configuration, which is macroscopically isotropic, thus the constitutive response can be expressed in terms of the stress and strain invariants. An incremental model which follows the evolving yield surface must take into account the anisotropic nature of the evolving microstructure. This results in a more complex structure of the constitutive model. In many practical powder metallurgical manufacturing procedures elements of material see smoothly changing monotonically increasing stress states, allowing the use of a simple deformation theory model.

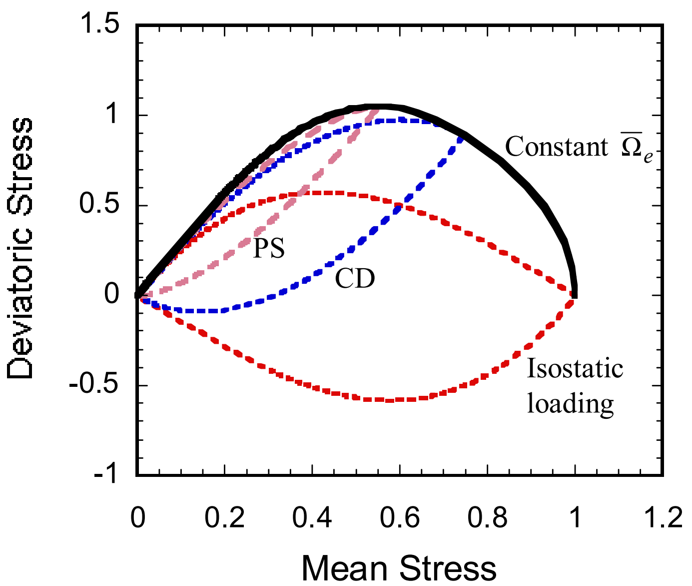

Figure 2 Yield surfaces for the particulate material of Figure 1generated under isostatic, pure shear (PS) and simulated closed die compaction (CD) which nest inside the surface of constant $\overline{\Omega}_e$.

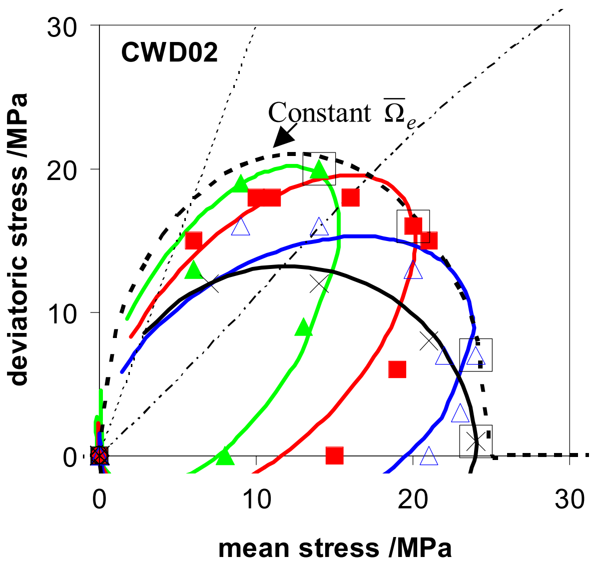

Figure 3 Experimental yield surfaces and a surface of constant $\overline{\Omega}_e$ for a commercial steel powder during the early stages of compaction.

References

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