MODELING MARTENSITIC TRANSFORMATION IN THE ELASTO-PLASTIC MATERIAL AT FINITE STRAIN

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Summary In this paper, a model of martensitic transformation in TRIP steel was established in the framework of the continuous mechanics and thermodynamics at a large strain. The model is based on the concept of a laminated microstructure composed of the martensitic plate and austenite layer. The internal structure of the martensite and austenite composite is variable and changes with moving interface. The model includes the essential features of the deformation induced martensitic transformation and provides a local kinetics description of martensite growth. A distinctive feature of the current model is that each phase is characterized by its own material constitutive model, and therefore, the evolution of the stress in both phases as the martensite transformation proceeds under a given deformation gradient can be properly predicted.

INTRODUCTION

The aim is to develop a physically-based, multi-scale model to predict the structure-property relations in low alloyed multi-component TRIP steel. It is suggested that a unique combination of high ductility and high toughness in this steel come from both TRansformation Induced Plasticity (TRIP) and the synergy between the properties of multicomponents [1]. Therefore, the model should include the martensitic transformation, the plastic flow of the matrix phases, the plastic flow of the parent phase and the interaction between the transformed region and its surroundings. A general scheme of the multi-scale model is demonstrated in Figure 1. As a first step, this paper presents a model for deformation-induced martensitic transformation at the micro-level in the framework of the continuous mechanics and thermodynamics.

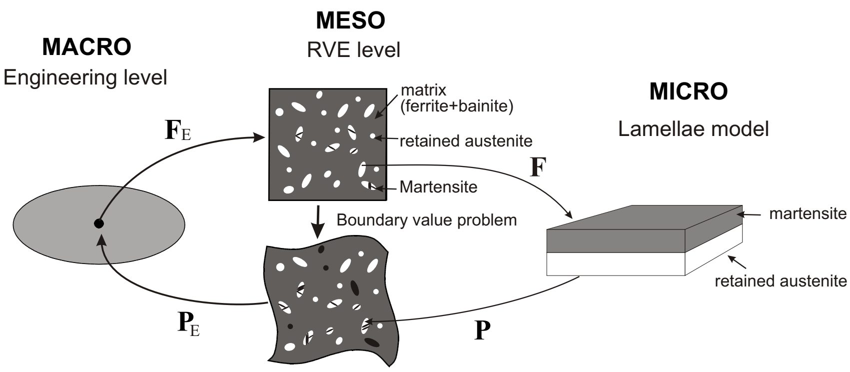

Figure 1. A general scheme of the multi-scale model for multi-component low alloyed TRIP steel.

KINEMATICS OF MARTENSITIC TRANSFORMATION AND DEFORMATION

A major feature of the martensitic transformation is its shape deformation, which can be observed as a surface relief effect and has the characteristics of an invariant-plain strain. According to the phenomenological theory of the martensite crystallography [2], the shape deformation, represented by total transformation deformation gradient, $\mathbf{F_{tr}}$, can be described by:

$$\mathbf{F_{tr}} = \mathbf{I} + \vec{M} \otimes \vec{N} \tag{1}$$

where $\vec{N}$ is the habit plane unit normal and $\vec{M}$ is the shape deformation vector, $\mathbf{I}$ is the second order unit tensor.

Let the motion of a material point in a process of martensitic transformation be described by the deformation gradient tensor $\mathbf{F}$. We assume that the deformation gradient in the austenite can be multiplicatively decomposed into an elastic part, $\mathbf{F^e_A}$, and a plastic part, $\mathbf{F^p_A}$. After martensite transformation it is possible to decompose the total deformation gradient into a plastic deformation gradient $\mathbf{F^p_A}$ in austenitic phase, a transformational $\mathbf{F_{tr}}$ and an elastic and a plastic deformation gradients, $\mathbf{F^e_M}$ and $\mathbf{F^p_M}$ in martensitic phase. Therefore,

$$\mathbf{F_A} = \mathbf{F^e_A} \cdot \mathbf{F^p_A} \qquad\qquad \mathbf{F_M} = \mathbf{F^e_M} \cdot \mathbf{F^p_M} \cdot \mathbf{F_t} \cdot \mathbf{F^p_A} \tag{2}$$

THE MARTENSITIC TRANSFORMATION CRITERION

The local transformation criterion proposed by Fischer and Reisner [3] and Levitas [4] is applied as:

$$G = [\phi] - \rho_0^{-1} \cdot \langle \mathbf{P} \rangle^{\mathbf{T}} : [\mathbf{F}] \geq G_c \tag{3}$$

where $\langle \mathbf{P} \rangle = (\mathbf{P_A} + \mathbf{P_M})/2$ with $\mathbf{P_A}$ and $\mathbf{P_M}$ being the first Piola-Kirchhoff stress tensors in the austenite and martensite phases, respectively, $[\mathbf{F}] = \mathbf{F_A} - \mathbf{F_M}$ is the jump of the deformation gradient across the moving interface, $[\phi]$ is the jump of the Helmholtz free energy across the interface, ρ_0 is the mass density of the austenite. G is the effective driving force including both chemical and mechanical terms, acting at the material points on the moving interface, which can be obtained by applying the jump condition across the interface. G_c is the threshold value of the transformation barrier, which includes the contributions from both phase transformation and plastic dissipation. It depends on not only the shape deformation strain but also the history of the stress and strain variation during the transformation process. In this research we assume that G_c is a material constant.

MODEL FOR DEFORMATION-INDUCED MARTENSITE TRANSFORMATION

The total deformation gradient tensor $\mathbf{F}$ at a transforming region is assumed to be known. As soon as the martensite forms, this microregion can be treated to consist of austenite containing parallel layers of martensite plate. The parallel interface separating the two constituents are characterized by the habit plane normal $\vec{N}$ in the reference configuration. As martensitic transformation is an invariant plane strain deformation, therefore,

$$\mathbf{F_A} \cdot (\mathbf{I} - \vec{N} \otimes \vec{N}) = \mathbf{F_M} \cdot (\mathbf{I} - \vec{N} \otimes \vec{N}) \tag{4}$$

Assuming a quasi-static equilibrium state, the balance of linear momentum on the interface requires that:

$$\mathbf{P_A} \cdot \vec{N} - \mathbf{P_M} \cdot \vec{N} = 0 \tag{5}$$

The deformation and stress fields within each phase (layer) are assumed to be homogeneous. The given deformation $\mathbf{F}$ and the first Piola-Kirchhoff stress $\mathbf{P}$ at the transforming region is assumed to be distributed between the phases according to the rule of mixtures

$$\mathbf{F} = (1-\xi)\mathbf{F_A} + \xi \mathbf{F_M} \qquad \mathbf{P} = (1-\xi)\mathbf{P_A} + \xi \mathbf{P_M} \tag{6}$$

Where ξ is the volume fraction of the martensite. Each phase is characterized by its own constitutive relations. A finite strain logarithmic elasto-plastic constitutive relations [5] is assumed for austenite while elastic behavior is taken for martensite

$$\boldsymbol{\tau}_\mathbf{A} = f(\mathbf{C_A}, \ln \mathbf{B^e_A}, \triangle\gamma) \qquad \boldsymbol{\tau}_\mathbf{M} = \frac{1}{2}\mathbf{C_M} : \ln \mathbf{B^e_M} \tag{7}$$

Where $\mathbf{B^e_A}$ and $\mathbf{B^e_M}$ are the elastic left Cauchy-Green deformation tensors in austenite and martensite, respectively; $\boldsymbol{\tau}_\mathbf{A}$ and $\boldsymbol{\tau}_\mathbf{M}$ are the Kirchhoff stress tensors in austenite and martensite, respectively. $\mathbf{C_A}$ and $\mathbf{C_M}$ are the fourth-order tensors of material constants; $\triangle\gamma$ is the increment of plastic flow in austenite. The standard J_2 plasticity model restricted to isotropic hardening is applied to the austenite phase.

By solving system of equations constructed by (2)-(7), the martensite volume fraction and the stress evolution in each phase during martensite growth can be obtained. The increment of the martensite appeared ($\Delta\xi$) is calculated in such a way that $G = 0$ (where G is calculated with the updated values of the variables, e.g.$\mathbf{P}$, $\mathbf{F}$ etc.). If the calculated fraction $\Delta\xi$ is less than zero, no transformation occurs during this increment, otherwise, if $\Delta\xi > 0$, the transformation takes place. This algorithm is analogous to the return-mapping algorithm in plasticity.

CONCLUSIONS

A model of deformation-induced martensitic transformation in TRIP steel was established in the framework of the continuous mechanics and thermodynamics at a large strain. The model provides a local kinetics description of martensite growth and the evolution of the stress in both phases as the martensite transformation proceeds.

References

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