INCREMENTAL EFFECTIVE CONSTITUTIVE LAW FOR COMPOSITE MATERIAL IN THE FORM OF ARTIFICIAL NEURAL NETWORK

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Summary Description of effective behaviour of composites in the form of a suitably trained Artificial Neural Network is presented in this paper. We assume an incremental form of constitutive relationship approximated by ANN. We propose three methods of acquisition of data for training the ANN. The first one furnishes the pairs: average stress - average strains and their increments - resulting from the solution of boundary value problem defined over a representative volume, with given mean strain imposed. The second and the third methods base on sampling of global behaviour of the composite material. The presented approach is fully numerical. The corresponding algorithm seems to be applicable for a large class of composites. The representation of the constitutive law by ANN can be included as a subroutine into any FE code. Example shows the application of the method for Neo-Hookean material.

INTRODUCTION

For a non-linear composite an adequate description of effective behaviour is usually very difficult to obtain. The classical, symbolic constitutive law is usually theoretically deduced from known properties of a representative volume, basing on a suitable version of homogenisation theory. In this paper we use an Artificial Neural Network (ANN) trained with pairs: mean stress – average strain (or respective increments) has been used as a numerical representation of the effective constitutive law. Different strategies of collecting the training data are possible. Three of them are analysed in the paper.

FORMULATION OF THE PROBLEM

Let $\mathbf{u}^0$, σ^0 be the solution of the "homogenised" problem i.e. a classical BV problem of mechanics in which the variable material coefficients $\boldsymbol{a}(\mathbf{y})$ in:

$$\sigma^{\varepsilon}_{ij}(\mathbf{u}^{\varepsilon})(\mathbf{x},\mathbf{y}) = a_{ijkl}(\mathbf{y})e_{kl}(\mathbf{u}^{\varepsilon}(\mathbf{x},\mathbf{y})) \quad \text{(and } \sigma^{\varepsilon}_{ij} \in P(\mathbf{y}) \text{ where is an admissible subset of of the stress space)}$$

are replaced with some unknown constant values $\boldsymbol{a}^h$. We suppose that the periodicity of material characteristics imposes an analogous periodical perturbation on the quantities describing the mechanical behaviour of the body. Hence, for displacements we have:

$$\mathbf{u}^{\varepsilon}(\mathbf{x}) \equiv \mathbf{u}^0(\mathbf{x}) + \varepsilon\,\mathbf{u}^1(\mathbf{x},\mathbf{y}) \;,\; \mathbf{u}^1 \text{ Y-periodic}$$

For $\boldsymbol{a}^h$ uniquely defined via classical homogenisation procedure $\mathbf{u}^{\varepsilon}$ converges weakly to $\mathbf{u}^0$ as ε tends to zero. In this case the tensor $\boldsymbol{a}^h$ defines an effective constitutive relationship between $\mathbf{e}(\mathbf{u}^0)$ and σ^0 (a new admissible set in the stress space P^h makes a part of this effective constitutive description). In this paper, instead of looking for $\boldsymbol{a}^h$ and P^h we are going to use a numerical representation of the effective constitutive law by a suitably trained ANN N. The input-output vector of N contains components of strains $\mathbf{e}(\mathbf{u}^0)$ and stress tensor σ^0 (in the vector-like notation):

$$\boldsymbol{\sigma}^0 = N @ \mathbf{e}(\mathbf{u}^0): \quad \sigma_i = \sum_m w^2_{q(i)} f_{(q)}\left(e_p w^1_{p(q)} + \theta^1_{(q)}\right) + \theta^2_{(i)}$$

Weights, biases and transfer functions are respectively $\mathbf{w}$, θ and f in (4). The finite element code with N included as a material description subroutine can be used to solve for $\mathbf{u}^0$ the problem:

$$\text{find } u^0_i \in V \text{ and } \sigma^0_{ij} \in L^2 \text{ be such that: } \forall\; v_i(\mathbf{x}) \in V$$

$$\int_{\Omega h} \sigma^0_{kl}(\mathbf{u}) v_{k,l}\, d\Omega = \int_{S_f} F_i v_i\, dS_f \qquad \boldsymbol{\sigma}^0 = N^h @ \mathbf{e}(\mathbf{u}^0)$$

The correct, effective constitutive law in the above, represented by the ANN called N^h assures the following requirement:

$$\forall \mathbf{F}: \left\| \mathbf{u}^{\varepsilon} - \mathbf{u}^0 \right\|_{L^2(\Omega)} \leq \tau \qquad \tau \leq \varepsilon \left\| \mathbf{u}^1 \right\|_{L^2(Y)} \quad \text{as } \varepsilon \text{ tends to } 0$$

The above expression defines a correspondence between the heterogeneous and the homogeneous (homogenised) fictitious body in the manner suitable for our purpose. In the numerical practice the comparison of the effective solution with the exact one is checked only in few strategic points. The above equivalence condition formulated for any **F** is verifiable in practice only for a finite number of prescribed loads. In the example below the only one case of load is reported. In what concerns the convergence to zero with cell's diameter: as it is shown in many papers, a relatively small trying portion of the composite is sufficient to predict the overall behaviour of the non-homogeneous material.

THREE STRATEGIES

The most natural method of data acquisition assumes that ANN is trained with pairs: mean stress – average strain (or respective increments). The averaging is done over a cell of periodicity or a representative volume of the composite. The second method of data acquisition uses a composed neural network (CNN). Inside the CNN a Hopfield-Tank sub-network specialises in construction of a variety of statically admissible fields for a fictitious, "homogenised" body. An ANN with hidden layer associates the generated, statically admissible stresses with strains computed from the displacements, known from a numerical experiment. The third method we use in the training process is a so called "self-learning" finite element procedure, developed recently by Shin and Pande. Hybrid, FE-ANN code generates some trial solutions obtained for an initial approximation of the constitutive law. These trial solutions are then corrected after a comparison with known behaviour of the studied composite. According to the "self-learning" method displacements are compared and the difference (error) is then used – via FE solution of the new BV problem - to improve the initial guess. In the second and third methods the data are collected from many points of the sample. The composition: CNN and ANN with hidden layers - can be used for elaboration of results of the experiment in which homogeneous state of stress is not needed. Because of this a large class of data resulting from observations of real structures can be used as a source of knowledge about the structural constitutive behaviour. Representation of a Constitutive Law by ANN with Hidden Layers is crucial for the proposed modelling procedure. In the paper, We discuss the structure of the network, its approximation abilities. The networks automatic generalisation capability enables us to predict the material behaviour i.e. to produce the graph stress-strain for an arbitrary sequence of stress or strain values, never presented to the ANN before.

EXAMPLES

We show two examples: regular, perpendicular mesh of elastic-plastic inclusions in elastic-plastic matrix and the second – hiper-elastic material with a regular array of circular holes. The first example was studied by analysis of 20 cells of periodicity, the second – by a single, repetitive cell.

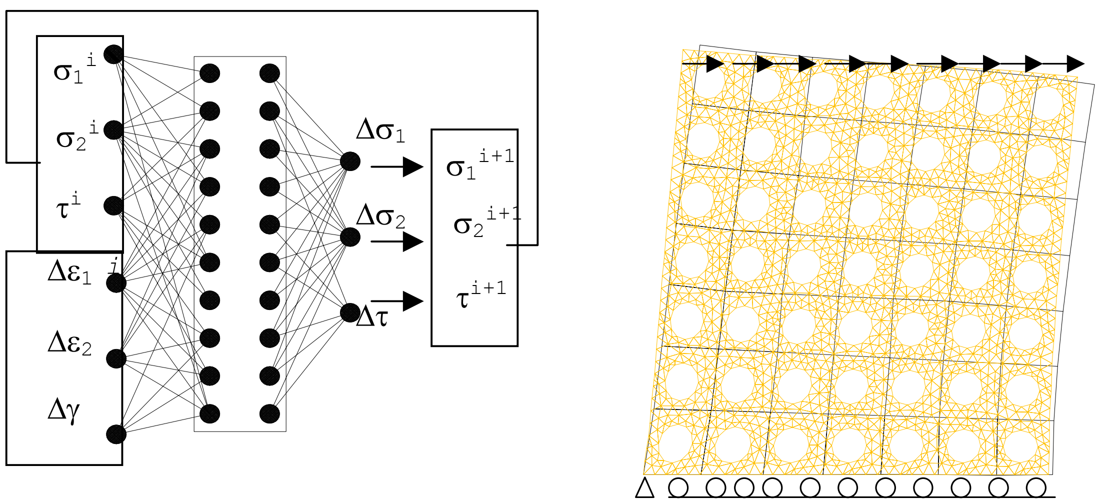

Figure 1.a Scheme of the ANN for approximation of a constitutive operator and autonomous networks activity in computing σ along a given path in space of strains. 1.b. Comparison of displacements: FE model with hyperelastic material (light lines), FE code with ANN defining the homogeneous material (coarse rectangular mesh scale of the displacements is 1:1). This case of loading has never been presented to the network in training

CONCLUSIONS

The ANN representation of any constitutive law is a flexible tool for identification of the global (effective) behavior of materials with complex internal structure. This representation is derivable from both possible sources of knowledge about the material: real experiment and numerical simulation. ANN representation is very suitable for analysis of composites since it is "automatic" in the sense that it does not require any "a priori" choice or adaptation of the existing constitutive theory for the description of the observed material behavior. We show that it is possible to incorporate the ANN constitutive description into a Finite Element code. A realistic FE model can be thus constructed for a material described by ANN. The examples show that the model is possible even in the case of complicated non-linear behaviour. Convergence of the third method is surprisingly fast. Three or four steps are enough to obtain a qualitatively good model.