EVALUATION OF LINEARIZATION PROCEDURES SUSTAINING NONLINEAR HOMOGENISATION THEORIES

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Summary A systematic methodology for the evaluation of various existing linearisation procedures sustaining mean field theories for nonlinear composite materials is proposed and applied to the 'modified secant' and various 'affine' formulations. It relies on the analysis of a composite for which an exact treatment of both the nonlinear homogenisation problem and the linear homogenisation of the 'linear comparison material' with identical microstructure is possible; the effects of the sole linearization method can then be evaluated.

Introduction

The present study aims at defining a methodology and a numerical tool which could be used for a systematic evaluation of the various existing homogenization methods for nonlinear heterogeneous materials and composites. These methods classically rely on two separate procedures:
- first, they propose a way to approximate the actual nonlinear behavior by linear constitutive equations, through a specific linearization procedure, so as to define a heterogeneous linear comparison material ('LCM');
- an appropriate linear homogenization model is then chosen to predict the overall as well as the local responses of this fictitious LCM.

A number of such nonlinear homogenization schemes have been proposed recently which deserve systematic comparisons both at the global and the local levels in order to select the most efficient ones or to propose improved new ones. When doing so, two main difficulties have to be overcome: first the two above-mentioned procedures (i.e., linearization and linear homogenization) should preferably be evaluated separately in order to focus attention on the most critical step of defining the LCM and dealing with the intra-phase heterogeneity; second, the comparison would be more conclusive if an exact and complete (i.e. providing both global response and local fields) solution of the studied problem was available.

Proposed methodology

These statements have led to the choice of a periodic microstructure so as to make possible an exact (numerical) resolution of the nonlinear problem and, after application of various linearization approaches, to be left with a linear homogenization problem to be addressed with the same (periodic) microstructure, which can also be solved exactly. Thus, attention can be focused only on the specific effects of the choice of such or such linearization procedure through the comparison of the associated predicted overall properties and local stress and strain fields with the exact solution. Such a methodology improves on more classical approaches, in which predictions of mean field theories for nonlinear random microstructures are often compared to numerical simulations for periodic composites, the effects of the linearisation procedures and the approximate treatment of the homogenisation of the LCM being then combined.

As a first step, constitutive behavior deriving from one single potential only is considered. For the sake of simplicity, the studied material reduces to a (periodic) two-phase composite with a nonlinear matrix (constitutive relation $\sigma = f(\epsilon)$ and aligned identical elastic spheroidal inclusions. The periodic unit cell is subjected to periodic conditions and an axial loading. The nonlinear solution is obtained thanks to a finite element computation under code Cast3M, whereas the linearization procedures (incremental, classical and modified secant methods, classical affine, with its full formulation as well as several simplified formulations, are considered) are implemented in C++ routines. The effective properties and the local responses of the LCM are derived exactly by means of an additional FEM Cast3M computation, which is interfaced with the C++ code.

Application

An illustrative example of preliminary results already obtained this way is described below. It is first devoted to the evaluation of the so-called 'modified secant formulation' proposed by Suquet [1], in close connection with Ponte Castañeda's variational procedure [2], through the quantitative comparison of its global and local (not shown here) predictions with the exact ones. According to this formulation the local behavior of the constitutive phases of the LCM is isotropic and defined by $\sigma(x) = L^{sct}(\langle\epsilon\otimes\epsilon\rangle) : \epsilon(x)$ with $L^{sct} = 3kJ + 2\mu^{sct}(\langle\epsilon\otimes\epsilon\rangle)K$, where μ^{sct} is the secant moduli. The second-order moment of the local strain in the matrix $\langle\epsilon\otimes\epsilon\rangle$ can be evalutated by direct integration of the numerical solution, or by means of a partial derivative of the overall axial response with respect to the local shear moduli. Only one axisymmetric FEM calculation is required for the LCM even though its overall linear behavior is anisotropic. The same analysis has also been applied to the classical 'affine' linearisation procedure [3, 5] for which constitutive equations of the matrix in the LCM read $\sigma(x) = L^{tgt}(\langle\epsilon\rangle) : \epsilon(x) + \tau^0(x)$, with use of the anisotropic tangent moduli $L^{tgt} = \frac{df}{d\epsilon}(\langle\epsilon\rangle)$ and the eigenstress $\tau^0 = f(\langle\epsilon\rangle) - L^{tgt}(\langle\epsilon\rangle) : \langle\epsilon\rangle$. In the case of uniaxial load, the local elastic behavior in the LCM exhibits transverse isotropy : $L^{tgt} = 3kJ + 2\mu^{sct}(\langle\epsilon\rangle)F + 2\mu^{tgt}(\langle\epsilon\rangle)E$ - notations of [4], and so does its overall linear behavior. The 'full anisotropic implementation' of the affine formulation requires the computation of all the components of the effective tensor of moduli and that is why transverse and longitudinal shearing modes must be considered through a Fourier analysis in addition to the axial symmetric modes (tension and compression in the transverse plane). This complex formulation can

be simplified first by considering an isotropic matrix with moduli given either as $L^m = 3kJ + 2{\mu_m}^{tgt}(\langle\epsilon\rangle)K$ (simplified affine-1), or $L^m = 3k_m J + 2{\mu_m}^{iso}(\langle\epsilon\rangle)K$ where ${\mu_m}^{iso} = \frac{4{\mu_m}^{sct} + {\mu_m}^{tgt}}{5}$ (simplified affine-2), but still computing exactly the anisotropic behavior of the LCM. An additional simplification is provided when, in addition, one assumes the overall behavior isotropic and identifies its moduli through its response to tension ('full isotropic' affine formulation).
All the corresponding results are shown in Figure 1.

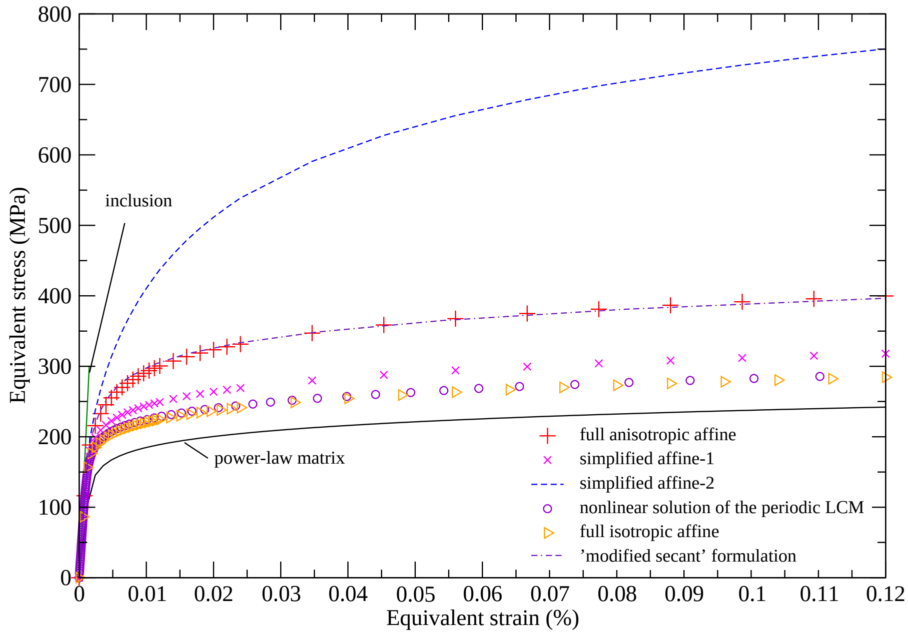

Figure 1. Effects of the choice of a linearization method (modified secant and various affine versions) on the tensile stress-strain curve of a two-phase composite. Inclusion volume fraction : 30%; Inclusion elastic properties : $E^I = 400$ GPa and $\nu^I = 0.3$. Power-law matrix properties: $E^m = 75$ GPa, $\nu^m = 0.2$, $n = 10$, $\epsilon_0 = 1\%$ and $\sigma_0 = 300$ MPa.

Conclusions

As a preliminary conclusion, one can notice that the 'full isotropic' affine linearization treatment yields the softest global response, and is very close to the exact nonlinear solution. This apparently good prediction of this very approximate procedure is probably the consequence of two opposite trends: the 'classical' formulation of the affine procedure, which does not account for intra-phase heterogeneities in the definition of the LCM, is known to yield somewhat too hard predictions and this is compensated by the recourse to a very soft simplification of the tensor of tangent moduli (${\mu_m}^{tgt} << {\mu_m}^{sct}$). The deeper analysis of this comparison, in particular in terms of local strain and stress fields, but also local density of potential energy, is under progress and should provide more insight. The evaluation of the recently proposed 'second-order' linearization procedure [4], closely related to the affine one [5], but taking into account to some extent the intra-phase heterogeneity, is also planed and might lead to other conclusions regarding the pertinence of the recourse to a simplified isotropic tensor of tangent moduli. More generally, it is expected to propose from such comparisons some optimized linearization treatment, with respect to some criteria, still to be defined. In the long term, the application of such a methodology to two-potential behavior might also be investigated.

References

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