Fuzzy Set Approach to Modelling Composite Mechanical Properties

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Introduction

The field of composites mechanics, engineering and technology is relatively young, and the test methods and measurements techniques are not yet fully developed. The modelling of their mechanical properties is even further behind the experimental investigations. The study and application of composite materials is a truly interdisciplinary endeavour since their properties are strongly dependant on the physical and chemical bonding between three constituents: a reinforcement, a matrix and an interface layer being in fact a new material. Therefore, the origin and source of imprecision (or uncertainty) lies mainly in the lack of information dealing with their microstructure, mechanical properties of constituents, behaviour and the number of factors responsible for gradual degradation of their properties and final failure. Commonly, the theoretical (deterministic) analysis of composites is based on homogenisation theories that may include an increasing number of different parameters. However, it is unknown in advance what number of parameters is sufficient to describe satisfactorily the problem considered. On the other hand, the material parameters are evaluated in the experimental way being the source of randomness in the traditional (deterministic) analysis or impreciseness or vagueness in the fuzzy set approach. The imprecise, vague, qualitative, linguistic or incomplete information may be present in geometry, material properties, degradation of properties, applied loads or boundary conditions.

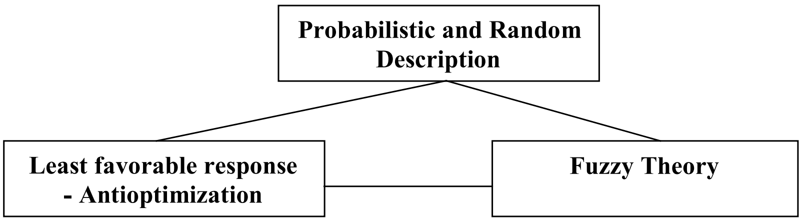

Fig.1 Approaches to modelling uncertainty

In the analysis of engineering problems three different approaches are used depending on the nature and extent of uncertainty by the introduction of the uncertainty triangle (Muc [1]) – see Fig. 1. If we know the probability distributions of the system variables and the covariance matrices the performance of the system may be considered with the use of probability theory. If only the fragmentary information on the uncertainty quantity is available an upper bound on the maximum response may be evaluated using the anti-optimisation approach. Between the above models lies the third approach – the fuzzy set theory that can be implemented to predict the structural response in the sense of evaluation of its upper and lower bounds, respectively.

In the paper we intend to present qualitative and quantitative differences of results (understood in the sense of composite mechanical properties) comparing two models: a deterministic and the second, which is based on the fuzzy, set approach. They are used in the description of two types of composite materials – a unidirectional and a textile (a twisted yarn composite). In both cases considered herein the fuzziness of variables is expressed by the triangular membership function. The variability (understood as the fuzziness) of material and geometrical parameters is taken to be equal to ±10 %, whereas the nominal (average) values correspond to α=1 and are equivalent to the deterministic description. The vertex method associated with the α-cuts representation of the membership functions is used to derive the upper and lower bounds of fuzzy output variables [1].

Micromechanics model of unidirectional composites

The Aboudi micromechanics model [2] is applied to study the effects of micromechanics constituent properties as fuzzy variables. In the model the unidirectional composite is represented by fibres, which are aligned in the x_1–direction and distributed regularly in the matrix and finally the components form a doubly periodic array in the x_2 and x_3 directions. The fibres have a rectangular cross-section (h_1, l_1) and are arranged at distances h_2, l_2 apart. The cell is further divided into four subcells, each of them with a local coordinate system. At the micromechanics level, the transversely isotropic material is expressed in terms of the five engineering constants. They are functions of the material variables, i.e. of the fibre and the matrix properties: $E_{f11}, E_{f22}, G_{f12}, \nu_{f12}, \nu_{f13}, E_m, \nu_m, V_f$,where V_f is the fibre volume ratio, *f* denotes fibre property, and *m*–a matrix property. Let us assume that the above variables characterising the fibre and matrix properties, the fibre volume fraction and the representative cell geometry are fuzzy variables. Using the procedure proposed above for a given α -cut one can evaluate the upper (the right end point) and lower bounds (the left end point) of the effective transverse Young modulus. The numerical results are evaluated for the glass/epoxy resin, i.e. the identical material as discussed in Ref. [3]. The lower and upper bounds plotted in Fig. 2 (denoted by the superscripts L and R) are not symmetrical with respect to the nominal curve (α =1).

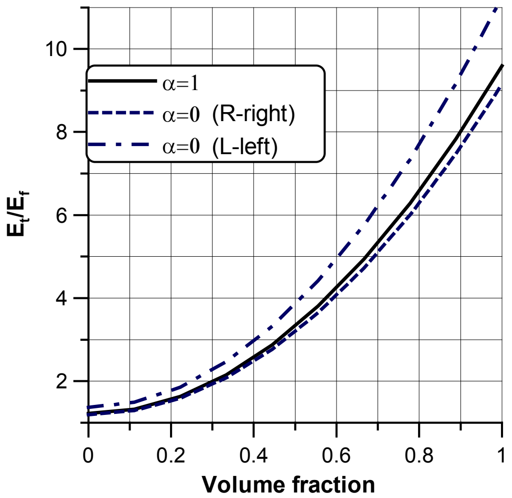

Fig.2 Distributions of the effective transverse Young modulus at α = 0 and 1 for unidirectional composites

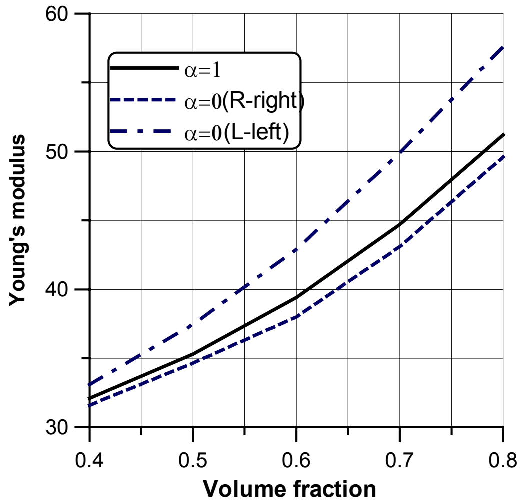

Fig.3 Variations of Young's modulus with fibre volume fraction for twisted yarns

Fuzziness in textile composites

The development of innovative fibre architecture and textile manufacturing technology has significantly expanded the potential of fibre reinforced composites. An emerging area is textile composites reinforced with 3-D preforms such as woven, braided and knitted fabrics. The integrated fibre network provides stiffness and strength in the thickness direction thus reducing the potential for the interlaminar failure. To demonstrate the impreciseness in the modelling the elastic constants of twisted yarn composites are predicted based upon the unit cell structure and volume averaging of elastic constants of constituent materials – see Ref. [4]. Similarly as previously the composite material is assumed to have transversely isotropic properties.

The geometry of twisted yarns is characterised by the twist angle θ and the number of yarns twisted expressed by a parameter λ – linear density of a twisted yarn. Geometrical parameters θ, λ and Young's moduli of fibres and matrix have been treated as the fuzzy parameters. However, the fibre volume fraction is assumed to be constant as the geometrical parameters of the twisted yarn vary. The nominal properties, both material and geometrical, are identical to those prescribed in Ref. [4]. The results of computations are plotted in Fig.3. As it may be seen, again there is no symmetry with respect to the nominal curve denoted by the value α =1 (the solid line).

Concluding Remarks

The effectiveness of the proposed method can be estimated by the comparison with experimental results. Therefore, it is interesting to note that the presented in Ref. [5] comparison of the theoretical and experimental data shows that the experimental values are located rather above the nominal curve (the solid line in Fig.2) what directly corresponds to the obtained fuzzy results. For textile composites similarly as in the case of the unidirectional fibres the experimental data lie above the nominal curve corresponding to the theoretical deterministic predictions – Ref [4].

At the end of this section it is worth mentioning that the presented method allows one to build the appropriate membership functions for the evaluated values of the Young moduli taking into account various (not only triangular) normalised to 1 probabilistic distributions of mechanical properties of constituents and their geometrical ratios. In this way it is possible also to take into consideration the interface layer having variable, unknown in advance, mechanical and geometrical properties.

References

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