PLANE HARMONIC WAVES IN A MICROPERIODIC LAYERED THERMOELASTIC SOLID REVISITED

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Summary A one-dimensional dynamic coupled thermoelastic refined averaged theory for a microperiodic composite is used to study plane harmonic waves in a layered infinite solid. In such a theory an eight-order in time partial differential equation involving a high intrinsic mechanical frequency Ω_M and a high intrinsic thermal frequency Ω_T is a central one. It is shown that if Ω_M and Ω_T are finite, there are two harmonic thermoelastic waves of a given frequency ω that propagate in a positive direction normal to the layering. Numerical results illustrating propagation of the two waves in a nanoperiod composite are included.

INTRODUCTION

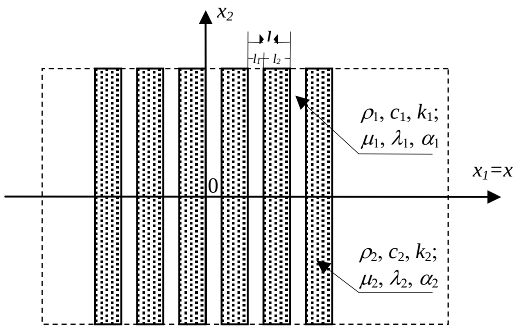

Fig. 1 A microperiodic layered infinite thermoelastic solid

Consider a layered infinite thermoelastic solid which is composed of an infinite number of identical thin subunits that form a spatially periodic pattern with a period *l* as shown in Fig. 1. Each subunit consists of two layers that, in general, have different dimensions and are made of different homogeneous isotropic thermoelastic materials. Let $l_i, \rho_i, c_i, k_i, \mu_i, \lambda_i$, and α_i (i=1,2), respectively, denote the thickness, density, specific heat, thermal conductivity, shear modulus, Lame modulus, and thermal expansion of the *i*th layer in a subunit. Also, assume that an external thermomechanical load is uniformly distributed over a plane parallel to the layering for every time $t > 0$. Then a thermoelastic process corresponding to the load is one-dimensional, and can be described by the dimensionless partial differential equations (see [1]-[2])

$$\left\{\left(\frac{\partial^2}{\partial\tau^2}+\kappa_1^2\right)\left[\left(\frac{\partial^2}{\partial\xi^2}-\frac{\partial}{\partial\tau}\right)\left(\frac{\partial}{\partial\tau}+\alpha\right)-(\alpha-\beta)\frac{\partial^2}{\partial\xi^2}\right]+\varepsilon_1\left(\frac{\partial}{\partial\tau}+\alpha\right)\frac{\partial^3}{\partial\tau^3}\right\}$$
$$\times\left[\left(\frac{\partial^2}{\partial\tau^2}+\kappa_1^2\right)\left(\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\tau^2}\right)-\kappa_2^2\frac{\partial^2}{\partial\tau^2}\right]S-\varepsilon_2\left(\frac{\partial^2}{\partial\tau^2}+\kappa_3^2\right)^2\left(\frac{\partial}{\partial\tau}+\alpha\right)\frac{\partial^3}{\partial\tau^3}S=0 \qquad (1a)$$

$$\left(\frac{\partial^2}{\partial\tau^2}+\kappa_3^2\right)\frac{\partial^2}{\partial\tau^2}\theta-\left[\left(\frac{\partial^2}{\partial\tau^2}+\kappa_1^2\right)\left(\frac{\partial^2}{\partial\xi^2}-\frac{\partial^2}{\partial\tau^2}\right)-\kappa_2^2\frac{\partial^2}{\partial\tau^2}\right]S=0 \qquad (1b)$$

where $S = S(\xi,\tau)$ and $\theta = \theta(\xi,\tau)$ denote the stress and temperature fields, respectively; the parameters κ_1, κ_2, and κ_3 represent the frequencies proportional to an intrinsic mechanical frequency Ω_M while ε_1 and ε_2 stand for the thermoelastic coupling parameters; and α and β represent the frequencies proportional to an intrinsic thermal frequency Ω_T .

Equation (1a) is an eight-order in time partial differential equation that plays a central role in the theory. Once a solution $S = S(\xi,\tau)$ to Eq. (1a) is found, the temperature $\theta = \theta(\xi,\tau)$ is obtained by integrating Eq. (1b) with respect to time.

The existence of two time-periodic solutions (S,θ) to Eqs. (1) that represent waves propagating in a positive direction normal to the layering was established in [1]-[2] when $\Omega_M \to \infty$ and $\Omega_T < \infty$ or $\Omega_M < \infty$ and $\Omega_T \to \infty$. In the present paper the existence of two harmonic thermoelastic waves is proved when $\Omega_M < \infty$ and $\Omega_T < \infty$. In the next section an existence theorem on two harmonic thermoelastic waves is formulated, and a numerical analysis of the two waves for a particular composite is presented. Finally, results and conclusions are summarized.

PLANE HARMONIC WAVES IN A MICROPERIODIC LAYERED THERMOELASTIC SOLID WHEN $\Omega_M < \infty$ AND $\Omega_T < \infty$

We look for a solution to Eq. (1a) in the form

$$S(\xi,\tau) = \hat{S}\exp[i(\omega\,\tau - \eta\xi)], \quad i^2 = -1, \quad |\xi| < \infty, \quad 0 \le \tau < \infty \qquad (2)$$

where $\hat{S}$ is a constant, ω is an assigned frequency $(\omega > 0)$ and η is the wave number to be selected from the condition that $S = S(\xi, \tau)$ be a nontrivial solution to Eq. (1a). A function *S* given by Eq. (2) represents a plane harmonic stress wave propagating in the ξ-direction with a velocity *c* and with an attenuation *q* if

$$\eta = \omega / c - i\, q, \quad c > 0, \quad q > 0 \tag{3}$$

Substituting $S = S(\xi, \tau)$ from Eq. (2) into Eq. (1a) a fourth-degree algebraic equation for η is obtained. A root analysis of this equation leads to the Theorem: If $0 < \omega < \kappa_3(l) < \kappa_1(l)$ and $0 < \varepsilon_2 < 0.5$ then there are two wave numbers of the form

$$\eta_k = \omega / c_k - i\, q_k, \quad c_k > 0, \quad q_k > 0, \quad (k = 1, 2) \tag{4}$$

The associated thermoelastic waves are then represented by the formulas

$$S_k(\xi, \tau) = \mathrm{Re}\,\{\hat{S}_k \exp[i(\omega\, \tau - \eta_k \xi)]\} \tag{5}$$

$$\theta_k(\xi, \tau) = \mathrm{Re}\,\{\hat{S}_k \omega^{-2} (\kappa_3^2 - \omega^2)^{-1} [(\eta_k^2 - \omega^2)(\kappa_1^2 - \omega^2) - \kappa_2^2 \omega^2] \exp[i(\omega\, \tau - \eta_k \xi)]\} \tag{6}$$

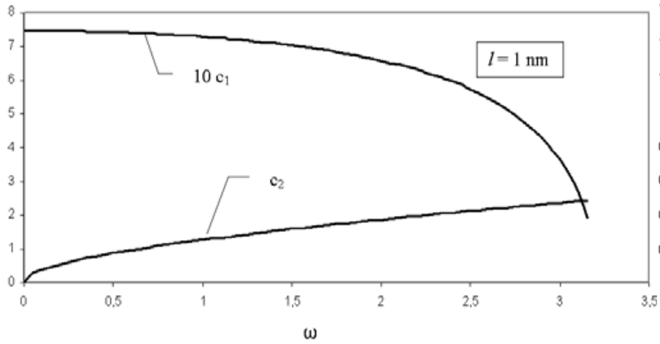

Fig. 2 The velocities c_1 & c_2 treated as functions of ω for a nanocomposite.

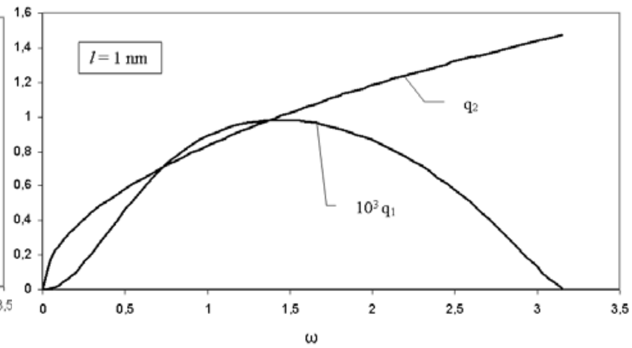

Fig. 3 The attenuation coefficients q_1 & q_2 treated as functions of ω for a nanocomposite

where $\hat{S}_k$ stands for a constant and $\mathrm{Re}\,\{\cdot\}$ denotes the real part of $\{\cdot\}$. To illustrate numerically the solution (4)-(6) we assume that each subunit is made of a zirconium oxide and a titanium alloy layers (see [3]). Figures 2 and 3, respectively, show the pairs (c_1, c_2) and (q_1, q_2) treated as functions of ω for l = 1nm. It follows from Figures 2 and 3 that a thermoelastic wave propagating with the smaller velocity is almost dispersionless and of a small damping, while that of the greater velocity reveals a high dispersion and a large damping. An analysis of the wave profiles S_1, θ_1, S_2 and θ_2 propagating along the positive ξ-axis indicates that the pair (S_1, θ_1) is represented by an isothermal elastic wave with a small damping $(\theta_1 \approx 0)$ while (S_2, θ_2) is represented by a thermoelastic wave with a fast decay on the ξ-axis.

CONCLUSIONS

- A one-dimensional averaged theory of thermoelastic waves in a microperiodic layered infinite solid is revisited in which an eight-order in time PDE involving a high intrinsic mechanical frequency Ω_M and a high intrinsic thermal frequency Ω_T is a central one. It is shown that if Ω_M and Ω_T are finite, there are two harmonic thermoelastic waves of a given frequency ω that propagate in a positive direction normal to the layering.
- The numerical analysis of the two waves for a zirconium oxide-titanium alloy composite with a period l = 1nm indicates that a wave propagating with the smaller velocity is almost dispersionless and isothermal in nature while that of the greater velocity reveals a high dispersion and a large damping.
- The two harmonic thermoelastic waves may be useful in a study of harmonic waves in a microperiodic layered semi-infinite thermoelastic body.

References

[1] J. Ignaczak, Plane harmonic waves in a microperiodic layered infinite thermoelastic solid, *Proceedings of The Fifth Int. Congress on Thermal Stresses;TS2003*, June 8-11, 2003, Blacksburg, VA, USA, pp. TM-1-1-1 to TM-1-1-4.

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