MATERIAL INSTABILITIES IN THERMO-MECHANICAL PROCESSES

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Summary The paper investigates the roles of thermal effects and thermo-mechanical couplings in the development of material instabilities and strain localisation. The main result is that during a thermo-mechanical process, two conditions are shown to play the essential roles and correspond to the singularity of the isothermal and the adiabatic acoustic tensors.

INTRODUCTION

Thermal effects have important consequences on behaviour of materials and are fully accessible to measurements. For instance, experimental results relative to polymers demonstrate that fracture initiation and growth may be influenced by temperature variations ahead of the crack tip . Thermal loading strongly affects the plastic deformation of soils and rocks and consequently strain localization in these materials . In structural steel, the important effect of heating on nucleation and growth of shear bands has been observed. These experimental evidences suggest that thermal phenomena can play an important role in promoting or inhibiting global and local instabilities in materials. The objective of this work is to fully include these thermal effects and thermo-mechanical couplings in the analysis of material instabilities.The main result r evidences that adiabatic analyses may be not sufficient to understand the development of material instabilities and strain localization phenomena.
Different thermal effects and thermo-mechanical couplings affect the mechanical behaviour of solids. In particular, while it is known that all mechanical properties are usually temperature dependent and constitute concurrent causes of thermal softening, other less recognized phenomena such as thermal expansion, heat conduction, mechanical dissipation and phase transformations need to be considered.

THERMOINELASTIC CONSTITUTIVE EQUATIONS

The analysis is carried out for a broad class of *rate-independent*, coupled thermo-irreversible constitutive equations including thermoplasticity and thermodamage. The presentation here is restricted to small strain for simplicity. In the framework of continuum thermodynamics of irreversible processes, the behaviour of the material is described by:

- The Helmholtz free energy per unit mass $\Psi = \Psi(\boldsymbol{\epsilon}, \boldsymbol{\alpha}, T), \quad \Psi = e - Ts$ where $\boldsymbol{\epsilon}$ is the (small) strain tensor, T the absolute temperature and $\boldsymbol{\alpha}$ is a generic collection of internal variables of various tensorial nature (scalars, vectors or second-order tensors) describing different physical mechanisms governing inelastic deformation. Moreover, e and s are the specific internal energy and the specific entropy, respectively.

- The reversibility domain, defining the range in which inelastic processes are excluded, is defined through the yield (or damage) function $f(\mathbf{A}, \boldsymbol{\alpha}, T) \leq 0$ where $\mathbf{A}$ are the thermodynamical forces associated to the internal variables $\boldsymbol{\alpha}$. Inelastic deformations are therefore possible only if $f = 0$, and during plastic flow the evolution of the internal variables must satisfy Prager's consistency $\dot{f} = 0$.

- The evolution of internal variables given by $\dot{\boldsymbol{\alpha}} = \dot{\lambda}\mathbf{P}, \quad \mathbf{P} = \frac{\partial F}{\partial \mathbf{A}}$ where the potential function $F = F(\mathbf{A}, \boldsymbol{\alpha}, T)$ is a function of the state variables and $\dot{\lambda}$ satisfies the Kuhn-Tucker conditions $\dot{\lambda} \geq 0, \quad f \leq 0, \quad \dot{\lambda} f = 0$

- Local conservation of energy or the First Law of Thermodynamics is $\rho \dot{e} = \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\epsilon}} - div \mathbf{q} + r$ where r is the heat supply per unit volume generated by internal sources, $\mathbf{q}$ is the heat flux, taken to obey a generalized Fourier law of heat conduction $\mathbf{q} = -\mathbf{K}\, \nabla T$ in which tensor $\mathbf{K} = \mathbf{K}(\mathbf{A}, \boldsymbol{\alpha}, T)$ is assumed positive definite.

Two constitutive properties are important in the analysis of material instabilities. These are the isothermal tangent modulus $\mathbf{H}^i$ and the adiabatic tangent modulus $\mathbf{H}^a$. The isothermal modulus relates the stress rate to the strain rate *under isothermal conditions* ($\dot{T} = 0$) while the adiabatic modulus relates the stress rate to the strain rate under *local adiabatic conditions* ($r = \div bq = 0$). These moduli are easily obtained when the the free energy Ψ, the yield function f and the potential F are given.

RESULTS

Within the above constitutive framework , strain localization under quasi-static conditions is addressed, using the classical shear band analysis restricted to the loading branch of the constitutive operator (Hill, 1962, Rice, 1976), though now extended to include thermal effects and thermomechanical couplings. This extension is performed by employing, additionally to the usual discontinuity conditions on mechanical variables, the jump conditions associated to the thermal variables, essentially, the temperature rate and the heat flux. This is done under general thermo-mechanical conditions in contrast to Raniecki (1976) who considered only the adiabatic situation. Localization is shown to first occur when either of the thresholds corresponding to singularity of the isothermal or adiabatic acoustic tensor is reached. The mechanical interpretation of such a condition involves the formation of a discontinuity of the heat flux divergence across a band in the former case or of the temperature rate in the latter case.

Plastic acceleration waves are also considered to show that propagation velocity is related to the eigenvalues of the isothermal acoustic tensor, while the adiabatic acoustic tensor does not play a role. A consequence of this is that, while strain localization corresponds to stationarity of acceleration waves in the isothermal case, when thermal effects are considered, localization may occur in adiabatic conditions, with still propagating acceleration waves.

The above analysis is completed by a perturbation analysis where at a certain stage of a homogeneous deformation of an infinite body, an infinitely small perturbation is applied and the resulting perturbed motion studied. Assuming a linearized response around the homogeneous state, the conditions for unbounded growth of perturbation are derived. When inertia is neglected, the perturbation analysis corresponds to a bifurcation analysis in a sense much similar to the internal instabilities defined by Biot (1965). The growth condition is found to be (when the rate of growth η is large enough, i.e. when $|\eta| \to \infty$)

$$\eta N \det[\mathbf{n} \cdot \mathbf{H}^a \cdot \mathbf{n}] + k\xi^2 \det[\mathbf{n} \cdot \mathbf{H}^i \cdot \mathbf{n}] = 0 \tag{1}$$

where N is a material positive constant and k the coefficient heat conduction(isotropic heat conduction was considered here so that $\mathbf{K} = k\mathbf{1}$ with $\mathbf{1}$ the second order unit tensor). Note that to obtain (1) perturbations were taken in the form $\delta\mathbf{X} = \tilde{\mathbf{X}} exp[i\xi\mathbf{n} \cdot \mathbf{x} + \eta t]$ with ξ the wavenumber and η the rate of growth of the perturbation. $\tilde{\mathbf{X}}$ is the amplitude of the perturbation. It is therefore shown that for a quasi-static path —which in general is neither isothermal nor adiabatic— internal instability corresponds either to the singularity of the isothermal acoustic tensor (localization in the incrementally isothermal conditions) or to the singularity of the adiabatic acoustic tensor (localization in the incrementally adiabatic conditions) and is associated to the full range of wavelengths. Under dynamic conditions, unbounded growth is associated only to the short wavelength regime and to divergence growth or flutter phenomena governed by the *isothermal* acoustic tensor.

COMMENTS

The analyses performed in the present paper share analogies with those relative to deformation of porous, fluid-saturated inelastic materials, where pore pressure plays a role similar to heat conduction. Isothermal and adiabatic conditions are replaced in that context by drained and undrained conditions. However, differently from the porous plastic problem, the adiabatic constitutive tangent operator is not symmetric for thermoplasticity, even in the relevant case of associative flow rule. An important consequence of this is that a hierarchy between instability in incrementally isothermal and adiabatic conditions does not exist, so that the latter may be critical for stability.

The analysis was limited in the present study to a homogeneous and infinite solid. A current analysis shows that many of the results contained in this paper remain valid when one considers heterogeneous situations and includes boundary conditions. These boundary conditions however bring new features.

It may be also important to mention, in closure, that the obtained results may be of interest in the numerical treatment of coupled problems, with emphasis on the consequences of material instabilities on numerical analyses. When perturbations may grow unboundedly, the initial boundary-value problem becomes ill-posed and one should expect mesh and *time step* dependency of the results. Classical algorithms are seen to be not consistent as no convergence is obtained as the time step $\Delta t \to 0$.

References

[1] Biot, M.A.: Mechanics of incremental deformations, J. Wiley and Sons, New York 1965.

[2] Hill, R.: Acceleration waves in solids, *J. Mech. Phys. Solids*, **10**:1-16, 1962.

[3] Raniecki, B.: Ordinary waves in inviscid plastic media. In Mechanical Waves in Solids ", (Edited by Mandel, J. and Brun, L.) CISM Courses and Lectures n. 222, Springer-Verlag, Wien, (1976).

[4] Rice, J. R.: The localization of plastic deformation, In *Theoretical and Applied Mechanics*, (Edited by Koiter, W.T.) North-Holland, Amsterdam, 207-220, 1976.