MODELING OF SOLITARY IMPULSES IN A COMPOSITE MATERIAL USING WAVELET ANALYSIS

Kateryna V. Terletska
S.P. Timoshenko Institute of Mechanics, Nesterov str. 3, 03680 Kyiv, Ukraine

Summary An original method for description of the solitary wave evolution in a material with the microstructure (a composite) is presented. A peculiarity of the proposed technique consists in using wavelet-analysis on the base of "Mexican hat" wavelets (MH-wavelets). Elastic wavelets based solution permits to reveal the microstructure effects, namely the essential dependence of the distance, on which evolution is visible, on the characteristic size of microstructure, and to extend the class of investigated pulses.

INTRODUCTION AND PROBLEM STATEMENT

We study plane solitary waves that propagate in a material with the microstructure. The material is considered within the framework of the microstructure theory of two-phase mixture. Solitary wave propagating in a mixture is considered to be plane, so in the concerned problem of the wave motion two independent variables only are presented – spatial coordinate x and time coordinate t.

One assumes that solitary wave is produced by the action of an impulse to the composite material, and this impulse propagates in the material as the solitary wave, that initially coincides by shape with the initial one. Modeling process consists of many stages. The initial stages include: the material model choice, choice of the appropriate impulse type, mathematical formulation of the wave motion problem and the choice of the analyzing way of the problem. Together they form theoretical part of the modeling.

The next part is linked with the wavelet-analysis application as the up-to-date tool of the computational mechanics. Elastic wavelets were suggested in [5], and were first applied for the solitary wave evolution study in [4]. In contrast to the prior studies when the initial pulses had to satisfy rather strict conditions, it is shown that initial impulse shape can be an arbitrary form (it should only admit wavelet representation and be solitary one). It is correct, however, under the condition that the profile can be approximated by MH-wavelets that possess the main property of elastic wavelets – they are solutions of the wave equation system of the considered problem. The certain class of solitary waves that correspond to known in classical experimental mechanics impulses [1] is studied in the work. These impulses were observed in classical experiments with short-time loadings.

ANALYSIS OF THE SOLITARY WAVE PROBLEM USING MH-WAVELET

The concrete dispersive medium, namely, microstructure mixture of two linearly elastically deformed components is considered. The basic system of equations for plane waves in such a medium is [4-6]

$$\rho_{\alpha\alpha}u_{1,tt}^{(\alpha)} - a_{\alpha}u_{1,xx}^{(\alpha)} - a_3 u_{1,xx}^{(\delta)} - \beta\left(u_1^{(\alpha)} - u_1^{(\delta)}\right) = 0. \tag{1}$$

Next step consists in MH-wavelets family using.

MH-wavelets were first used in computer vision to detect multiscale edges. It was noted earlier [4,5] that MH-wavelet, under some restrictions, is one of the solutions of system (1). This wavelet and its Fourier transform can be written analytically as [2,3]

$$\psi(x) = \frac{2}{\sqrt[4]{\pi}\sqrt{3\sigma}}\left(\frac{x^2}{\sigma^2} - 1\right) e^{-\frac{x^2}{2\sigma^2}}, \qquad \hat{\psi}(\omega) = -\frac{2\sqrt{2\sigma^5}\sqrt[4]{\pi}}{\sqrt{3}}\omega^2 e^{-\frac{\sigma^2\omega^2}{2}}. \tag{2}$$

Approximation of the initial impulse using MH-wavelets.

Let us choose the impulse that follows impulse obtained in [1] (see Fig.1.) Since MH-wavelets family form the tight frame [3] then the approximation, has the form

$$f_{\varepsilon}(x) \approx \frac{1}{A}\sum_{j=-2}^{9}\sum_{k=-14}^{14} d_{j,k}\psi_{j,k}(x), \tag{3}$$

where $A \approx 3.409$ is a frame bound and only coefficients $\left|d_{j,k}\right| > 10^{-4}$ are used.

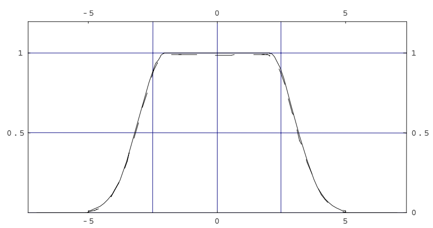

Fig.1

The idea of MH-wavelet application in the solitary wave evolution study lies in the representation of the initial profile using these wavelets and the assumption of the profile slow change during its propagation through weakly dispersive medium. Wavelet coefficients $d_{j,k}$ are supposed to be constant, and profile changes are results of nonlinear changes of phase velocities that are included in MH-wavelet arguments $v_{ph}^{(\alpha)}$. The idea of possibility of profile evolution descriptions by means of the phase variable only at the permanent functional profile representation lies in the basis of the simple wave theory [7]. Let us assume that the solution can be represented in the form of two displacements corresponding to two mixture phases and looking like initial impulse (3) with the wave phase in the argument

$$u^{(\alpha)}\left(x,t\right)=\frac{1}{A}\sum_{j=-2}^{9}\sum_{k=-14}^{14}d_{j,k}\psi_{j,k}\left(x-v^{ph}t\right).$$

This finite wavelet sum is the solution of wave equations (1) in the same meaning as MH-wavelets are solutions of (1) i.e. under some restrictions. The phase velocity, however, will depend on scaling level j.

Finally, impulse propagating in the mixture can be represented as

$$u^{(\alpha)}(x,t)=\frac{1}{A}\sum_{j=-2}^{9}\sum_{k=-14}^{14}\left(B^{(\alpha)}\tilde{f}_{(\alpha,l)}+p\left(z^{(\delta,j)}/l\right)B^{(\delta)}\tilde{f}_{(\delta,l)}\right), \tag{4}$$

$$\tilde{f}_{(\alpha,l)}=d_{j,k}2^{j/2}\psi\left(2^{j}\left(z^{(\alpha,j)}/l\right)-k\right), \qquad z^{(\alpha,j)}=x-v_{ph}^{(\alpha,j)}t.$$

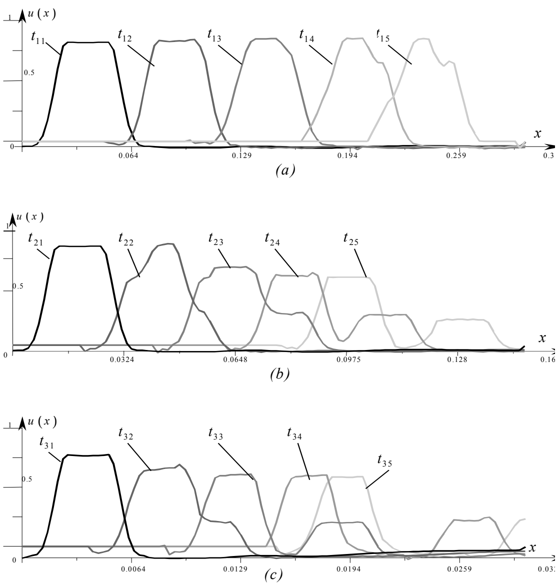

Fig. 2

Modeling of the solitary impulse evolution.

Composite materials “aluminum matrix - tungsten fibers” is chosen for the modeling. The volume fraction of tungsten fibers is $\xi_1 = 0,022$ [6]. Characteristic size of the microstructure is equal to $a^I = 0,648\cdot 10^{-3}m$. Waves with the length that exceeds the characteristic size of microstructure in 100, 50 and 10 times are examined. That is, the lengths of the wave bottoms are equal, respectively, to $0.648\cdot 10^{-1}, 0.324\cdot 10^{-1}, 0.648\cdot 10^{-2}m$. Amplitude coefficients are assumed to be $B^{(\alpha)} = B^{(\delta)} = 10^{-4}$. Profile evolution plots are built using analytical expression (4) and computation algorithms. Profiles are built for the first phase of composite material and for five sequential moments of time (in microseconds) $t_{11} = 25$, $t_{12} = 51$, $t_{13} = 76$, $t_{14} = 100$, $t_{15} = 120$, $t_{21} = 12,5$, $t_{22} = 25$, $t_{23} = 37,5$, $t_{24} = 50$, $t_{25} = 62,5$, $t_{31} = 2.5$, $t_{32} = 5.1$, $t_{33} = 7.6$, $t_{34} = 10$, $t_{35} = 12$ .Two main microstructural effects are well apparent. The first effect: is the breaking down the solitary wave into two modes that propagate with different velocities and simultaneous propagation of both modes in two composite components. As well, the second effect that concerns a microstructure is shown up. This effect consists in the essential dependence of the distance, on which evolution is observed, from the characteristic size of microstructure. So, the advantage of this method consists in the possibility of representation of wave impulses that haven’t analytic representations using elastic wavelets, which have analytic representation and are the solutions of the basic system of equations of plane waves in a composite material. It extends sufficiently the class of investigated impulses.

CONCLUSIONS

To solve the problem of solitary wave evolution during propagation through a material with the microstructure (a composite) the wavelet-analysis was applied. The main result consists, above all, in the ability of the proposed technique to study arbitrary wave shapes without any limitation on the initial profile. Also it permits to reveal the microstructure effects, and to extend the class of investigated initial pulses.

References

[1] Bell. J. F.: An experimental diffraction grating study of the split Hopkinson bar experiment. *J. Mech. Phys. Solids* **14**: 309-321, 1966.

[2] Geranin V.O., Pisarenko L.D., Rushchitsky J.J.: Wavelet Theory with the Elements of Fractal Analysis. Tutorial with 32 lectures. VPF Ukr INTEI, K 2002.

[3] Mallat S. A.: Wavelet Tour of Signal Processing. Academic Press, SD-NY-L 1999.

[4] Rushchitsky J.J., Cattani C., Terletska E.V.: Evolution of solitary wave in material with microstructure – experience of wavelet-analysis application. *Int. Appl. Mech* **40**, 2004.

[5] Rushchitsky J.J., Cattani C.: Solitary elastic waves and elastic wavelets. *Int. Appl. Mech* **39**: 741-752, 2003.

[6] Rushchitsky J.J.: Elements of Mixture Theory. Naukova Dumka, K 1991.

[7] Zaslavsky G.M., Sagdeev R.Z.: Introduction to Nonlinear Physics: From Pendulum to Turbulence and Chaos. Nauka, M 1988.