Nonlocal effects in micromechanics of functionally graded composites of random structure

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Summary. The general non-local integral equation involving the statistical averages of stresses in the composite and inclusions of random structure functionally graded composite is obtained and solved by three different methods: the quadrature method, the iteration method, and the Fourier transform method with subsequent comparative analysis. The different nonlocal effects are detected, some of them are fundamentally new.

For *statistically inhomogeneous* composites [or functionally graded materials (FGM)], the ergodicity fails and ensemble and volume averages do not coincide. The degenerate case of these materials is a random matrix composite for which the inclusions are located in a region bounded in some directions, although unrestrictedness of the domain of inclusion locations does not preclude statistical inhomogeneity. For example, any laminated composite materials with randomly reinforced by aligned fibers in each ply, are the statistically inhomogeneous material. The concept of clusters is similar to that of fractal structures, and the role of statistical descriptor can be treated by such parameters as cluster size, the fractal dimension, and the radius of gyration. The informative characteristics of the random configurations use statistical second-order quantities which examine the association fillers relative to other particle in an immediate local neighborhood of the reference filler. We will analyze so-called ideal cluster materials where the concentration of particles is a piecewise and homogeneous one within the areas of ellipsoidal clouds and composed matrix. In particular, in this paper we will consider a single particle cloud with the shape of a thick ply located in an infinite matrix with zero concentration of particles. For FGM where the concentration of the inclusions is a function of the coordinates ($\varphi(\mathbf{x}_i)\neq$const.), the micromechanical approach is based on the generalization of the multiparticle effective field method, previously proposed for statistically homogeneous (SH) random structure composites (see for references and details [1]). The nonlocal integral effective operator of elastic effective properties is estimated. The nonlocal dependencies of the effective elastic moduli as well as of conditional averages of the strains in the components on the concentration of the inclusions in a certain neighborhood of a considered point are detected.

The trivial generalization of the approach for statistically homogeneous composites [1] leads to the estimation of statistical average of strains inside the inclusions

$$\langle\sigma\rangle_i(\boldsymbol{x})=\mathbf{Y}(\boldsymbol{x})\,\langle\sigma\rangle(\boldsymbol{x})+\boldsymbol{\mathcal{K}}(\boldsymbol{x})\langle\sigma\rangle_i\,,\quad \boldsymbol{\mathcal{K}}(\boldsymbol{x})\langle\sigma\rangle_i=\int \mathrm{K}(\boldsymbol{x},\boldsymbol{y})[\langle\sigma\rangle_i(\boldsymbol{y})-\langle\sigma\rangle_i(\boldsymbol{x})]d\boldsymbol{y} \qquad (1)$$

as well as effective properties. One obtains the explicit representations of tensors $\mathbf{Y}(\boldsymbol{x})$ and $\mathbf{K}(\boldsymbol{x},\boldsymbol{y})$ as the function of conditional probability density of inclusion locations $\varphi(\mathbf{x}_j|;\mathbf{x}_i)$ and their mechanical properties. The particular cases of the nonlocal integral Eq. (1) was solved by three different methods: the quadrature method, the iteration method, and the Fourier transform method with subsequent comparative analysis. For the SH media ($\varphi(\mathbf{x}_i)=$ const., $\langle\sigma\rangle(\boldsymbol{x})\neq$const), the standard scheme of iteration and Fourier transform methods permit one to obtain the explicit representations for the nonlocal integral and differential operators, respectively, of any order describing overall effective properties as well as the stress concentration factor in the components. It is shown that the integral operator reduces to the differential one for sufficiently smooth statistical average stress fields and demonstrated the advantage of the iteration method over the Fourier transform method. For the FGM, an applicability of the Fourier transform method is questionable (in contrast with the iteration method). Only at first glance, the relation (1) is equivalent to the corresponding one obtained for the global effective properties in the framework of zero order approximation ($\mathbf{K}(\boldsymbol{x},\boldsymbol{y})=\mathbf{0}$). The main difference is that $\mathbf{Y}(\boldsymbol{x})$ and, therefore, $\boldsymbol{L}^*(\boldsymbol{x})$ depends on the parameters of the inclusion distribution not only at the point $\boldsymbol{x}$, but also in a certain neighborhood of that point leading to a so-called *nonlocal effect*, though, of course, the effective parameter $\boldsymbol{L}^*(\boldsymbol{x})$ are the local ones in the sense of nonlocal elasticity theory. The diameter of this region mentioned above is estimated as three times the characteristic dimension of the inclusions. As a result, a statistically inhomogeneous composite medium behaves like a macroscopically inhomogeneous medium with local effective modulus $\boldsymbol{L}^*(\boldsymbol{x})$ determined for a nonlocal distribution of the inclusions in a certain neighborhood of the point considered. Let us consider a strip model of ideal fiber cluster with probability densities $\varphi_1(\boldsymbol{x})=n$ and $\varphi_2(\boldsymbol{x}_1|\boldsymbol{x}_2)=ng(|\boldsymbol{x}_1-\boldsymbol{x}_2|)$ inside the thick ply $|\boldsymbol{x}|<a_w=16a$ (a=1 is the inclusion radius) and 0 otherwise. We will consider the volume fiber fraction inside the ply c=0.65, and two radial distribution functions $g(r)$ (step and nonstop functions, see [1]). The neglect of the binary interaction of inclusions for statistically homogeneous medium $n(\boldsymbol{x})$=const. reduces the formula for the effective elastic moduli to the analogous relation obtained by the Mori-Tanaka method which is invariant to the $g(r)$. Assume the matrix is epoxy resin which

contains identical circular glass fibers. As can be seen from Fig. 1 the use of the approach based on the quasicrystalline approximation (also called Mori-Tanaka (MT) approach) leads to an underestimate of the effective moduli L^*_{2222} compared to the more exact approximation of the MEFM which provides a good comparison with experimental data for statistically homogeneous media (see [2]). The stress concentrator factors found at the previous local evaluations permits one to estimate the stresses in the inclusions $\langle\sigma\rangle_i(x_2)$ (see Fig. 2). For the SH media, we will consider inhomogeneous

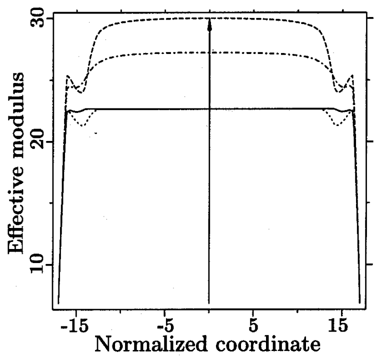

Fig. 1. L^*_{2222} vs x_2 estimated by: MEFM and nonstep $g(r)$ (dashed line), MEFM and step $g(r)$ (dot-dashed line), MT and nonstep $g(r)$ (dotted line), MT and step $g(r)$ (dotted line).

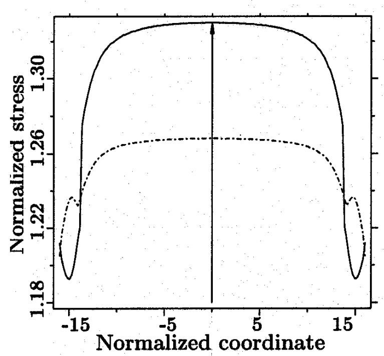

Fig. 2. $\langle\sigma_{22}\rangle_i(x_2)$ (solid line) and $\langle\sigma_{12}\rangle_i(x_2)$ (dot-dashed line) for the external loading at the infinity $\sigma^{\infty}_{ij}=\delta_{i2}\delta_{j2}$ and $\sigma_{ij}^{\infty}=\delta_{i1}\delta_{j2}$, respectively, and a step function g.

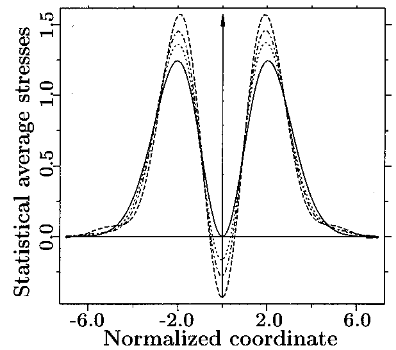

Fig. 3. $\langle\sigma_{22}\rangle_i(x_2)$ vs x_2 estimated for nonstep $g(r)$ by the MEFM. Zero order (solid line), first order (dotted line), second order (dot-dashed line), sevens order (dashed line) approximations.

loading $\langle\sigma_{ij}\rangle(\mathbf{x})=\delta_{i2}\delta_{j2}f(x_2)$ with the $f(x_2)=0.6584\,|x_2|^{1.999}\,e^{-0.2422\,x_2^2}$. The function $f(x_2)$ is not twice differentiable that makes the use of the Fourier transform method questionable. However, exploring of the iteration method for the solution of Eq. (1) presents no difficulties and provides fast convergence of the obtained iterations (see Fig. 3. and [3], [4]). It is noted that the estimation of the effective elastic moduli is a linear problem with respect to the local stress distribution analyzed which is less sensitive than nonlinear micromechanical problems of elastoplastic deformation, fracture, and fatigue of composite materials. However, the method also allows one to estimate the second moment of stresses in the constituents as well as at each point on the interface between the matrix and fibers. The dispersion of these interface stresses, defined only by stress fluctuations, will is used for the prediction of the effective envelope for failure initiation. The dependence of effective failure envelope on the elastic, geometrical, and failure parameters of the constituents and the interphase matrix/fibers are analyzed (see for details [5]). The nonelliptical shape of the effective failure envelopes (EFE) using the proposed method of integral equations is demonstrated in Fig. 4 (X=$\langle\sigma_{11}\rangle$, Y=$\langle\sigma_{12}\rangle$).

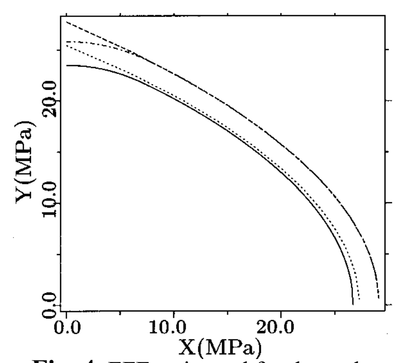

Fig. 4. EFE estimated for the real failure parameters for nonstep and step $g(r)$ (solid and dot-dashed curves) as well as for the assumed neglecting of shearing failure (dotted and dashed curves).

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