FRICTIONAL SLIP BETWEEN A GRADIENT NON-HOMOGENEOUS LAYER AND A HALF-SPACE IN ANTI-PLANE ELASTIC WAVE FIELD

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EXTENDED SUMMARY

The problem to treat here is shown As in Fig.1. An elastic layer with thickness H is pressed a elastic half-space by an applied pressure p°. Both the layer and half-space are composed of the gradient non-homogeneous medium, and suppose that there is the Coulomb friction with static and kinetic friction coefficients f_s and f_k at the interface between them The anti-plane motions in the layer and half-space will be aroused under the action of SH parabolic pulse incidence wave, as shown in Fig.2. When the shear elastic modulus $m(m^*)$ and the density $r(r^*)$ of medium vary by the laws of the second power in the gradient direction of medium, the analytical solution of the problem is found as

$$w=\int_{-\infty}^{+\infty}[A(k)(1+ay)^{\frac{1}{2}}H^{(1)}_{\frac{1}{2}}(p\frac{k}{a}(1+ay)) + B(k)(1+ay)^{-\frac{1}{2}}H^{(2)}_{\frac{1}{2}}(p\frac{k}{a}(1+ay))]e^{ikx}\,dk \tag{1}$$

$$w^*=\int_{-\infty}^{+\infty}[A^*(k)(1+a^*y)^{\frac{1}{2}}H^{(1)}_{\frac{1}{2}}(p^*\frac{k}{a^*}(1+a^*y)) + B^*(k)(1+a^*y)^{-\frac{1}{2}}H^{(2)}_{\frac{1}{2}}(p^*\frac{k}{a^*}(1+a^*y))]e^{ikx}\,dk \tag{2}$$

$w(w^*)$ is the anti-plane displacement; a, a^* are two known gradient constants of media; $p=\sqrt{\left(\frac{c}{c_s(c_s^\bullet)}\right)^2-1}$; c is the apparent velocity of elastic wave along the interface $c_s(c_s^\bullet)$ $c_s(c_s^\bullet)$ is shear wave speed of medium; A, B,A^*,B^* are known functions of k; asterisk * refers to the quantities of the layer; $H^{(1))}_{\frac{1}{2}}, H^{(2)}_{\frac{1}{2}}$ are Hankel function of 1,2 kind of 1/2 order. From these, he distribution of dimensionless shearing traction

$t(h)$ and that of dimensionless relative slip $d(h)$ are obtained as

$$
\begin{aligned}
t(h) = \mathrm{Re}\int_{-\infty}^{+\infty} & (H_{\frac{3}{2}}^{(2)}(p\frac{k}{a}) + \frac{A(k)}{A^{i}(k)} H_{\frac{3}{2}}^{(1)}(p\frac{k}{a})) \\
& \times \frac{2p(\frac{h}{l})}{pkH_{\frac{1}{2}}^{(2)}(p\frac{k}{a})} (\frac{\sin kl}{kl} - \cos kl) e^{ikx} dk
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
d(h) = \mathrm{Re}\int_{-\infty}^{+\infty} & (H_{\frac{1}{2}}^{(2)}(p\frac{k}{a}) + \frac{A(k)}{A^{i}(k)} H_{\frac{1}{2}}^{(1)}(p\frac{k}{a})) \\
& + \frac{A^{*}(k)}{A^{i}(k)} H_{\frac{1}{2}}^{(1)}(p^{*}\frac{k}{a^{*}}) + \frac{B^{*}(k)}{A^{i}(k)} H_{\frac{1}{2}}^{(2)}(p^{*}\frac{k}{a^{*}})] \\
& \times \frac{2(\frac{h}{l})}{pkH_{\frac{1}{2}}^{(2)}(p\frac{k}{a})} (\frac{\sin kl}{kl} - \cos kl) e^{ikx} dk
\end{aligned} \tag{4}
$$

Take $al = a^{*}l = 0.2$; $f_0 = f_k / f_s = 0.8$; $m^{*}/m = 1.5$; $ph/2l = 1$; $c^{*}/c = 1.2$; $q_0 = \sin^{-1}(c_s/c) = p/6$; $H/l = 50$; $x = (x - ct)/l$. $A^{i}(k)$ incident wave amplitude; Fig.3 shows the variation of $t(h)$ with h (real line) and the extent and location of the stick and slip zone for given parameters. It can be seen from the figure that the state of the interface reveals periodicity. The periodic length is 50. There are 3 slip zones (2 in right direction and 1 in inverse direction) in a periodic length, and on which put a number (1), (2) or (3), respectively. As the dimensionless external pressure s increase, the number of the slip zone will increase, its extent become larger. Fig.4 shows the distribution of the dimensionless shear traction $t(h)$ and dimensionless relative slip $d(h)$ in the slip zone (1)-(3) and neighborhoods of them. It can be seen from the figures that the shear traction $t(h)$ at the leading edge of the slip zone has a jump, but is continuous at the training edge, which is caused by the difference between the static and kinetic friction. Besides, there is a rigid displacement between the leading edge and the training edge. In order to reveal the effect of the interface thickness, we calculate the variations of the dimensionless shear traction t (33) and the dimensionless relative slip $d(33)$ with the dimensionless layer thickness H/l, as shown in Fig.5. From the figures we see that the change the dimensionless layer thickness H/l will make the state of the interface change, but the change is not monotonous.

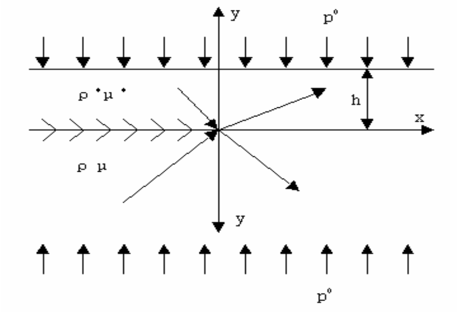

Fig.1 Anti-plane wave field in a half-space with a layer

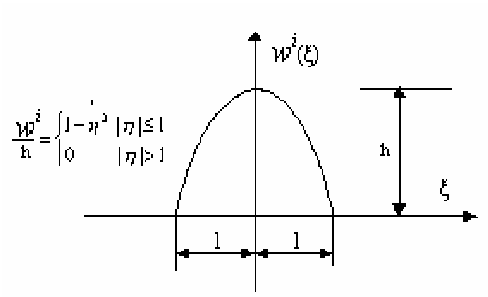

Fig.2 Incident parabolic pulse

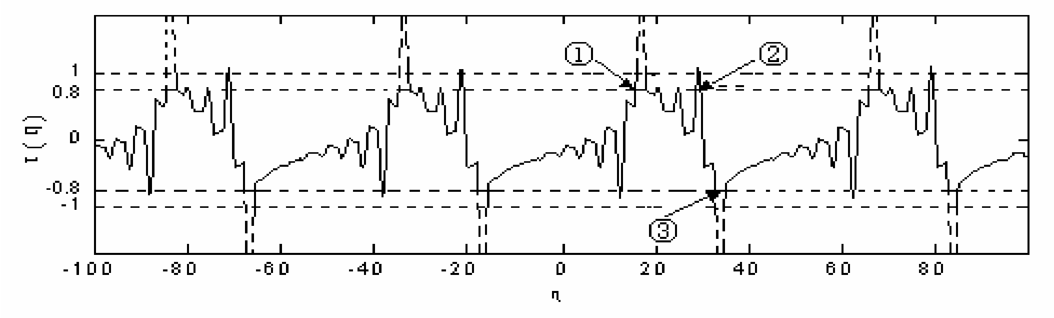

Fig.3 The stick and slip zones and distribution of shearing traction **τ (η)** at the interface

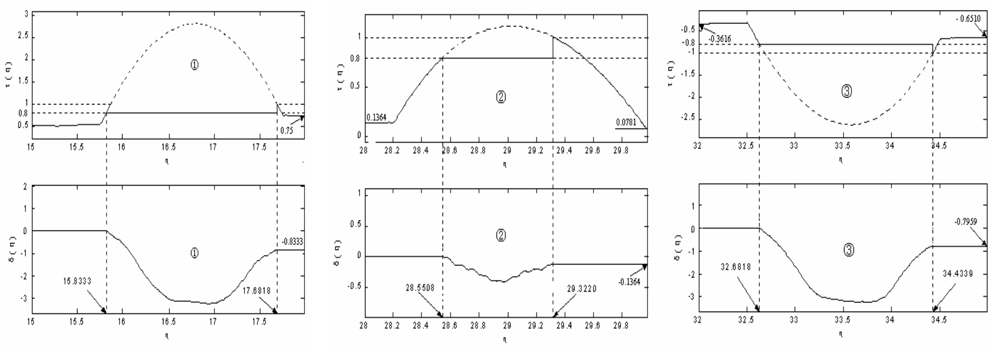

Fig. 4 Distribution of shearing tractions $t(h)$ **and relative slip** $d(h)$ at ① ② ③ point and its neighborhood

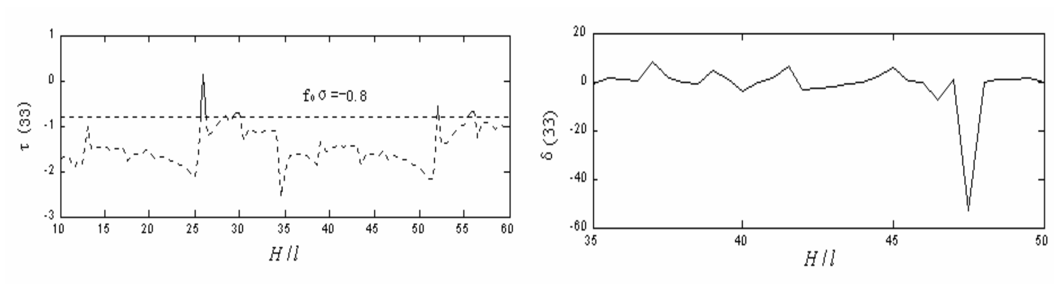

Fig.5 Variations of shearing traction $t(33)$ and relative slip $d(33)$ with dimensionless layer thickness H/l (real linear)

1. BaBaei R., Lukasiewicz S. A., 1998. Dynamic response of a crack in a functionally graded material between two dissimilar half planes under anti-plane shear impact load. Engineering Fracture Mechanics 60, 479-487.
2. BaBaei R., Lukasiewicz S. A., 1998. Fracture in functionally gradient material subjected to the time-dependent anti-plane shear load. ZAMM, 78, 383-390.
3. Comninou M. and Dundurs J., 1977.Reflection and refraction of elastic waves in presence of separation. Proceeding of Royal Society of London Series A356,509-528.
4. Comninou M. and Dundurs J., 1978.Elastic interface waves and sliding between two solids. Journal of Applied Mechanics 45,325-330
5. Comninou M. and Dundurs J., 1979.Interaction of elastic waves with a unilateral interface. Proceeding of Royal Society of London Series A368,1451-154.
6. Comninou M. and Dundurs J.,1980. Interface slip caused by an SH pulse. International Journal of Solids and Structure 16,283-289.
7. Comninou M. Barbe J.R.and Dundurs J.,1982. Disturbance at a frictional interface caused by a plane elastic pulse. Journal of Applied Mechanics 49,361-365.
8. Gai B.Z.,2000.Interaction of elastic waves with the unilateral interface between alayer and a half- space. International Journal of Solids and Structure 37,5233-5245.
9. Gai B.Z. and Chen C.C.,2001. BEM-analysis of interaction of elastic wave with unilateral interface crack. Communication in Numerical Methods In Engineering 17,137-147..
10. Liu. J.B., Wang. D., Yao. L., 1993. A contact force model in the dynamic analysis of contactable crack. Acta Mechanics Solida 4, 439-502. (in Chinese).
11. Li C. Y., Wang, G. J., 2001. Dynamic stress intensity factor of a cylindrical interface crack with a functionally graded interlayer. Mechanics of Materials 33, 325-333
12. Meguid S. A., Wang X. D., Jiang, L.Y., 2002. On the dynamic propagation of a finite crack in functionally graded materials, Engineering Fracture Mechanics 69, 1753-1768
13. Miller R.K.1977.Anapproximate method of analysis of the transmission of elastic waves through a friction. Journal of Applied Mechanics 44,625-656
14. Miller R.K.1978.The effects of boundary friction on the propagation of elastic waves. Bulletin of the Seismology Society of America 68,987-998.
15. Miller R.K. and Tan H.T.,1979. Reflection, refraction and absorption of elastic waves at a frictional interface: SH motion. Journal of Applied Mechanics 46,625-630
16. Miller R.K. and Tan H.T.,1981, Reflection refraction and absorption of elastic waves at a frictional interface :P and SV motion. Journal of Applied Mechanics 48,155-160.
17. Parameswaran. V. and Shukla A., 1999. Crack-tip stress fields for dynamic fracture in functionally gradient materials. Mechanics of Materials 31, 579-596.
18. Parameswaran. V. and Shukla A., 1998. Dynamic fracture of a functionally gradient material having discrete property variation. Journal of Materials Science 33, 3303-3311.
19. Shul C.W. and Lee K. Y., 2002. A subsurface eccentric crack in a functionally graded coating layer on the layered half-space under antiplane shear impact load. International Journal of Solids and Structure 39, 2019-2029..
20. Wang B.L., Han J. C and. Du S. Y., 2000. Crack problems in non-homogeneous composites subjected to dynamic loading, International Journal of Solids and Structure 37, 1251-1274
21. Wang Y.S. Yu G.L. and Gai B.Z., .1997.Slip with friction between an elastic layer and a substrate caused by an SH pulse. International Journal of Solids and Structure 35,2001-2021.
22. Wang Y.S. Yu G.L. and Gai ,B.Z.,1998.In-plane motion induced by an SH pulse at a frictional contact interface .Mechanic Research Communications25,203-210