DEFORMATION ANALYSIS OF INFLATED CYLINDRICAL MEMBRANE OF COMPOSITE MATERIAL WITH RUBBER MATRIX REINFORCED BY CORDS

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Summary An orthotropic hyperelastic constitutive model is proposed which can be applied to numerical simulation for the response of the nonlinear anisotropic hyperelastic material of the air-spring sheet used in inhibitive vibration of driver's seat and of the biological soft tissue. The parameters of strain energy function are fitted to the experimental results by the nonlinear least squares method. The deformation field of inflated cylindrical membrane of air-spring sheet is calculated by solving the system of five first-order ordinary differential and by the finite element method (FEM). Stability analysis is carried out in finite element analysis (FEA) to detect limit points by arc-length method.

INTRODUCTION

Recently, classical phenomenological constitutive equations for rubber-like solids, such as Mooney–Rivlin, Neo-Hookean or Ogden models (Holzapfel, 2000; Guo, 2001) are progressively replaced by more physical models based on statistical considerations in various engineering applications.

The main purpose of the authors is the numerical simulation of inflated cylindrical membrane of the composite made of rubber matrix reinforced by textile cords. The deformation field is determined by the method of the numerical integration of the system of the ordinary differential equations described recently for isotropic membrane by Guo (2001) and the FEM described by Shi-Moita (1996) and Verron et al (2001).

MATERIAL DESCRIPTION AND EXPERIMENTAL ANALYSIS

The material of air-spring sheet considered here is created from rubber matrix reinforced by textile cords to be called rubber-textile cord composite. The air-spring sheet is usually made up of four layers – the inner and the outer liner of calandered rubber and the two plies of cord reinforced rubber in which the cords have a specific bias angle to the other arranged symmetrically with respect to the circumferential direction. The cylindrical air-spring is relatively short – the diameter of the tubular sheet is 2R=80 mm, the height is L=120 mm and the wall thickness is H=2mm.

The experimental tests were carried out at five different positions of the air-spring. First the mounting plates of the non-loaded air-spring were fixed at the distance by 15, 20, 30, 40 or 50 mm shorter than the free height of the sheet. Then the air-spring was loaded and unloaded gradually by pressurized air step 0.05MPa in the range 0.1–0.5MPa. Photographs of the deformed sheet were recorded by digital camera, the axial force and the inner pressure were measured and stored at every stage of loading.

AN ORTHOTROPIC HYPERELASTIC CONSTITUTIVE MODEL

The orthotropic hyperelastic materials in this paper are considered incomprresible composite materials with two families of fibers. Let's assume that the isochoric deformation and neglect the dissipation due to irreversible effects. The strain energy function of these materials is considered the combination of scalar-value functions corresponding to energy stored in matrix material (isotropic) and the fibers (anisotropic) parts.

$$\Psi(\lambda_1,\lambda_2)=\sum_{i=1}^{3}\frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i}+\lambda_2^{\alpha_i}+\lambda_1^{-\alpha_i}\lambda_2^{-\alpha_i}-3\right)+\frac{k_1}{k_2}\left\{\exp[k_2(\lambda_2^2\cos^2\alpha+\lambda_1^2\sin^2\alpha-1)^2]-1\right\} \tag{1}$$

where λ_1 and λ_2 are the axial and circumferential stretches respectively, and the angle of the two families of reinforcing cords α is supposed to be 48.8^o. The parameters μ_i and α_i of Ogden's model of rubber (Holzapfel, 2000) are μ_1 = 630kPa, μ_2 = 1.2kPa, μ_3 = -10kPa, α_1 = 1.3, α_2 = 5, α_3 = -2. The stress-like parameter k_1 and the non-dimensional parameter k_2 are determined from the experimental results.

IDENTIFICATION OF MATERIAL PARAMETERS

The constitutive equations between Cauchy stresses and stretches are represented through strain energy function Ψ as

$$\sigma_1-\sigma_3=\lambda_1\frac{\partial\Psi(\lambda_1,\lambda_2)}{\partial\lambda_1},\ \sigma_2-\sigma_3=\lambda_2\frac{\partial\Psi(\lambda_1,\lambda_2)}{\partial\lambda_2}. \tag{2}$$

The quasi-static equilibrium equations applied to various inflated structures are

$$\frac{d}{ds}(T_1r)=T_2\frac{dr}{ds},\quad \kappa_1T_1+\kappa_2T_2=p\ . \tag{3}$$

where κ_1 and κ_2 are principal curvatures for the deformed membrane surface. T_1 and T_2 are the stress resultant forces per unit length of the meridional and circumferential directions: $T_1=h(\sigma_1-\sigma_3), T_2=h(\sigma_2-\sigma_3)$ (4)

From above constitutive equations and equilibrium equations after some transformations we obtain the system of nonlinear equations for the two variables k_1 and k_2. We solved this system of equations with experimentally measured values of λ_1 and λ_2 and obtained the parameters k_1 = 4.187e+04kPa and k_2 = -23.775.

DEFORMATION ANALYSIS OF INFLATED CYLINDRICAL MEMBRANE COMPOSITE

Determination of deformation of cylindrical membrane by numerical integration

After some substitutions and simplifications we obtained the system of five ordinary differential equations for the principal stretches λ_1 and λ_2, the tangent angle θ, the coordinate x in the deformed configuration and the inner pressure p with respect to the coordinate X of the undeformed configuration. We solved this system of differential equations by the shooting method in Matlab with the boundary condition determined from the experiments. The results are at the Fig. 1 where deformed profiles of membrane are compared with experimental one.

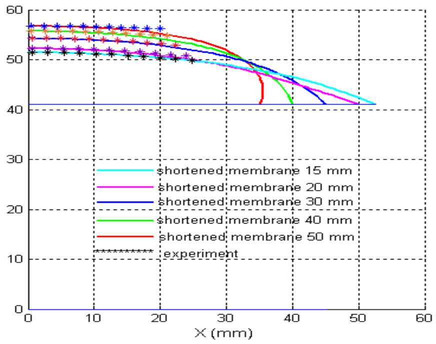

Fig.1 The deformed profiles of cylindrical membrane subjected to inner pressure p=0.5MPa

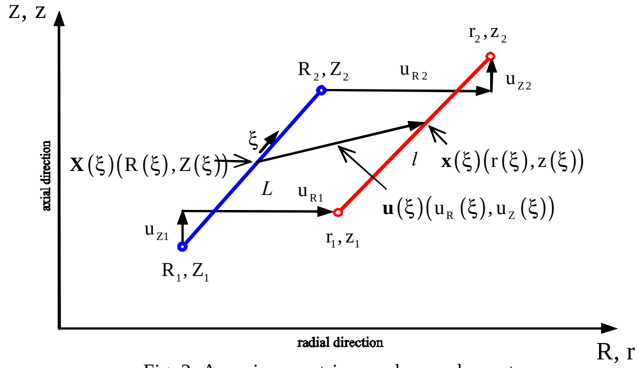

Fig. 2. An axisymmetric membrane element

Determination of deformation of cylindrical membrane by FEM

The axisymmetric membrane element that has length of L and thickness of H in the reference configuration and length of l in the current configuration is presented in Fig.2. This element has two nodes and vector node displacement $\mathbf{u}=\{u_{R1},u_{Z1},u_{R2},u_{Z2}\}^T$. Green-Lagrange (GL) deformation tensor $\mathbf{E}$ and second Piola-Kirchhoff (PK2) strain tensor $\mathbf{S}$ are used in the form of conjugate pair in order to express the formulation of strain energy in the Lagrangian description. The elastic tensor $\mathbb{C}$ is determined from principal components of PK2 (see Holzapfel, 2000, pp. 258).

$$\mathbb{C}=\frac{\partial \mathbf{S}}{\partial \mathbf{E}},\quad \mathbb{C}=\begin{bmatrix} \frac{1}{\lambda_1}\frac{\partial S_{11}}{\partial \lambda_1} & \frac{1}{\lambda_2}\frac{\partial S_{11}}{\partial \lambda_2} \\ \frac{1}{\lambda_1}\frac{\partial S_{22}}{\partial \lambda_1} & \frac{1}{\lambda_2}\frac{\partial S_{22}}{\partial \lambda_2} \end{bmatrix},\quad \mathbf{S}=\mathbb{C}\mathbf{E} \qquad (5)$$

The principle of virtual work can be expressed through introduction of the external load factor λ as

$$R(\mathbf{u},\delta\mathbf{u},\lambda)=\mathbf{R}(\mathbf{u},\lambda)^T\cdot\delta\mathbf{u}=(\mathbf{f}_{\mathrm{int}}(\mathbf{u})-\lambda\mathbf{f}_{\mathrm{ext}}(\mathbf{u}))^T\cdot\delta\mathbf{u} \qquad (6)$$

The stiffness matrix $\mathbf{K}_t$ is expressed in the following form: $\mathbf{K}_t=\left[\frac{\partial \mathbf{f}_{\mathrm{int}}}{\partial \mathbf{u}}-\lambda\frac{\partial \mathbf{f}_{\mathrm{ext}}}{\partial \mathbf{u}}\right]$ (7)

Nonlinear numerical solution: The incremental-iterative approach described here is based on a combination of the modified Newton-Raphson method and the arc-length method, meaning that the tangent stiffness matrix and internal force vector and external force vector are not only updated at the commencement of each load step, but also at every iterative cycles. Numerical simulated results are obtained by predictor-corrector method using Matlab software. The equilibrium path is obtained to overcome the limit points (Fig. 3 and Fig. 4).

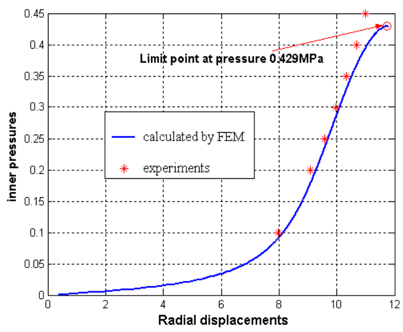

Fig.3 The internal pressure - radial displacement path in the case of free heads

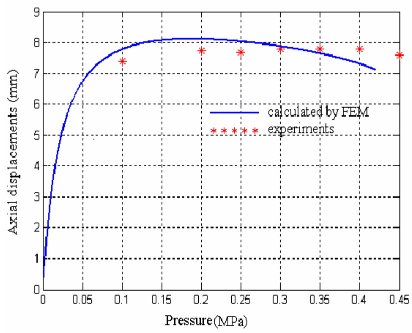

Fig.4 The internal pressure - axial displacement path in the case of free heads

CONCLUSIONS

The deformations of the nonlinear composite membrane were determined experimentally. The problem of the identification material parameters was solved. The deformations were determined by numerical integration based on the membrane theory and by FEM. Numerical results for simulation of the inflated cylindrical membrane are obtained answering by experimental responses.

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